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Notes on a Linear Formulation of Gravity

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Abstract

I expand upon my previous approach of formulating a linear theory of gravity in flat space-time. In particular, I present some additional field equations.

1 Introduction

I previously presented a formulation of gravity in which the background metric of space-time is flat [1]. Formulating a field equation in this flat background results in a linear theory, somewhat similar to Newton's theory, with a single field equation relating the stress-energy scalar to a tensor potential.The relevant equations are:

$$T\frac{8\pi G}{c^4} = -A \tag{1}$$

$$A_{\rho\sigma\mu\nu} = \frac{1}{2}(\Omega_{\rho\nu:\sigma\mu} - \Omega_{\rho\mu:\sigma\nu} + \Omega_{\sigma\mu:\rho\nu} - \Omega_{\sigma\nu:\rho\mu}) \tag{2}$$

$$A_{\sigma\nu} = N^{\rho\mu} A_{\rho\sigma\mu\nu} \tag{3}$$

$$A = N^{\sigma\nu} N^{\rho\mu} A_{\rho\sigma\mu\nu} \tag{4}$$

Note that there is a minus sign on the rhs of equation 1. This is a correction to the original paper [1], as will be apparent in the next section. Note that the symbols are defined as:

$T_{\mu\nu}$ = The stress-energy tensor.
$\Omega_{\mu\nu}$ = The tensor potential.
$\Omega_{\mu\nu:\rho}$ = Covariant differentiation of $\Omega_{\mu\nu}$ (:) with respect to the flat space-time.
$N_{\mu\nu}$ = The flat space-time backgound metric tensor.

The theory is a metric theory, in that matter follows geodesics of the combined metric:

$$g_{\mu\nu} = N_{\mu\nu} + \Omega_{\mu\nu} \tag{5}$$

We also have $\Omega = 0$ everywhere, and $A_{\sigma\nu} = 0$ in vacuum as demonstrated in the first paper [1].

2 Source Field Equation

I start off with the following manipulations for the point mass solution [1], as reproduced in the appendix. I use the fact that $\Omega = 0$ for this solution, and that colon (:) differentiation is commutative since the underlying space-time is flat

$$A_{\sigma\nu} = N^{\rho\mu} A_{\rho\sigma\mu\nu} = \frac{1}{2} N^{\rho\mu}(\Omega_{\rho\nu:\sigma\mu} - \Omega_{\rho\mu:\sigma\nu} + \Omega_{\sigma\mu:\rho\nu} - \Omega_{\sigma\nu:\rho\mu}) = \frac{1}{2} N^{\rho\mu}(\Omega_{\rho\nu:\sigma\mu} + \Omega_{\sigma\mu:\rho\nu} - \Omega_{\sigma\nu:\rho\mu}) \quad (6)$$

$$A = \frac{1}{2} N^{\sigma\nu} N^{\rho\mu}(\Omega_{\rho\nu:\sigma\mu} + \Omega_{\sigma\mu:\rho\nu}) = N^{\lambda\theta} N^{\rho\mu} \Omega_{\rho\lambda:\theta\mu} \quad (7)$$

We now define $B_\rho = N^{\lambda\theta}\Omega_{\rho\lambda:\theta}$ and apply the divergence theorem, recognizing that the covariant (:) derivative with respect to time is zero for this case, allowing us to ignore time. The only term that survives the surface integral over a sphere is the one where the index is the radial coordinate. We also note that we integrate over a large enough sphere that the source mass is effectively a point mass with respect to that sphere.

$$\oint_s B_1 \cdot \hat{r} ds = \oint_V N^{\rho\mu} B_{\rho:\mu} dV \quad (8)$$

We have:

$$B_1 = \frac{-\Omega_{11}}{r} = \frac{2GM}{r^2} \quad (9)$$

$$B_1 \cdot \hat{r} = \frac{-2GM}{r^2} \quad (10)$$

$$\oint_s B_1 \cdot \hat{r} ds = -8\pi G \oint_V \rho dV \quad (11)$$

Recognizing that the volume over which the integration is carried out is arbitrary, we have equation 1, after reintroducing the speed of light (c) which has been ignored, and recognizing ρ as being the stress-energy scalar.

3 Additional Equations

Note that I have tried all combinations of $\Omega_{\sigma\rho:\nu}, \Omega_{\sigma\nu:\rho}, \Omega_{\rho\nu:\sigma}, N^{\lambda\theta}\Omega_{\rho\lambda:\theta}N_{\sigma\nu}, N^{\lambda\theta}\Omega_{\sigma\lambda:\theta}N_{\rho\nu}, N^{\lambda\theta}\Omega_{\nu\lambda:\theta}N_{\sigma\rho}$ to create a 3-index tensor $B_{\rho\nu\sigma}$ that can be used in the divergence theorem to generate field equations analogous to the Einstein Field Equations, and have failed, but I have discovered the following tensor:

$$C_{\rho\nu\sigma} = \Omega_{\rho\sigma:\nu} - \Omega_{\rho\nu:\sigma} + N^{\lambda\theta}\Omega_{\sigma\lambda:\theta}N_{\rho\nu} - N^{\lambda\theta}\Omega_{\nu\lambda:\theta}N_{\rho\sigma} \quad (12)$$

The divergence of this tensor with repect to the first index is zero, proved by applying the divergence theorem to that index for the point mass solution and then using linearity to get the general solution. Expand the tensor out and put the first index as 1, confirming that it vanishes and thus has zero surface integral.

4 Conclusion

It appears that the tensor potential only depends upon the stress-energy scalar (Equation 1) and not the full stress-energy tensor. Additional constraints such as the tensor defined by Equation

12 having zero divergence, are expected to fully determine the tensor potential. We now state the following reasoning:

a) We assume that all matter follows geodesics with respect to the gravitational metric $g_{\mu\nu}$. This requires that the stress-energy tensor has zero divergence (with respect to the gravitational metric $g_{\mu\nu}$).

b) Einstein's equation may be seen as an initial value statement, saying that the universe was created in such a way that the stress-energy tensor was proportional to the Einstein tensor. Due to the zero divergence of the stress-energy tensor (with respect to the gravitational metric $g_{\mu\nu}$) Einstein's equation will remain true forever.

c) If a and b are true, then this theory is always consistent with Einstein's General Relativity. The linear theory presented here accounts for the production of the gravitational potential. The force law, which is non-linear, is encapsulated in Einstein's equation.

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6 References

[1] Wilson, Scott R. (2024), "An Alternative Formulation of Einstein's Field Equations of General Relativity", London Journal of Physics: Volume 1 No 1

7 Appendices

7.1 Solution For a Point Mass

$$\Omega_{\mu\nu} = \begin{bmatrix} -\frac{2GM}{r} & \frac{2GM}{r} & 0 & 0 \\ \frac{2GM}{r} & -\frac{2GM}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

7.2 Spherical Coordinates

For the present article, we will use spherical coordinates (r, θ, ϕ), with radial distance r, polar angle θ and azimuthal angle ϕ. With these coordinates the flat spacetime metric becomes:

$$N_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 sin^2\theta \end{bmatrix} \tag{14}$$

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The Christoffel symbols are:

$$\Gamma^1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & 0 & 0 & -rsin^2\theta \end{bmatrix} \tag{15}$$

$$\Gamma^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1/r & 0 \\ 0 & 1/r & 0 & 0 \\ 0 & 0 & 0 & -sin\theta cos\theta \end{bmatrix} \tag{16}$$

$$\Gamma^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/r \\ 0 & 0 & 0 & cos\theta/sin\theta \\ 0 & 1/r & cos\theta/sin\theta & 0 \end{bmatrix} \tag{17}$$

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