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Estimation of Energy Deposition via Detector Sections in Muon Scattering Tomography: Attestation Based on GEANT4 Simulations

A. Ilker Topuz

Manipal Centre for Natural Sciences, Centre of Excellence, Manipal Academy of Higher Education, Manipal 576104, India
Email: aitopuz@zohomail.eu

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Abstract

Traditionally, the Bethe-Bloch equation describes the parameters associated with the energy loss in course of the particle propagation in a certain medium. In this study, an attempt is made to determine the average deposited energy by means of the GEANT4 simulations for a set of materials including aluminum, copper, iron, lead, and uranium. The kinetic energy of the incoming muons on the detector layers is tracked, and the average energy deposited within the target material is computed by taking the difference between the kinetic energies at the detector layers. The simulation outcomes are compared and contrasted with those of Groom et al., 2001 by stating the corresponding statistics, i.e. the standard deviations. It is shown that the GEANT4 simulations might be utilized in order to calculate the mean deposited energy, the outcome of which is comparable with the existing data in the literature.

1 Introduction

The quantification of the energy loss by the charged particles as they traverse the matter stands as a fundamental aspect of the experimental particle physics or the related applied fields. Among the various charged particles, the cosmic ray muons are of particular interest due to their penetrating power and relatively well-known interaction mechanisms that make them ideal probes in the non-destructive imaging techniques such as muon scattering tomography [1–4]. The theoretical framework for describing the energy deposition of the relativistic charged particles is founded on the Bethe-Bloch equation that successfully predicts the stopping power across a wide range of materials and energies [5, 6].

Recent advancements in the computational methods, e.g. Monte Carlo simulations, have enabled the detailed modeling of particle interactions with the complex detector geometries and the heterogeneous target compositions. GEANT4, which is a widely adopted C/C++ framework for the simulation of the passage of particles through matter, offers a robust platform for studying the details of the energy deposition events [7]. Through its physics models as well as material databases, GEANT4 can provide the quantitative predictions that are directly comparable to the experimental measurements as well as the established reference data.

In this study, the estimation of the mean energy deposited by the cosmic ray muons in the selected target volumes is performed by using the GEANT4 simulations. The primary objective is the validation of the simulation results against the established values reported by Groom et al., 2001 [8], thereby attesting to the reliability of the computational approach for the practical applications in the muon scattering tomography. The methodology concerns recording the kinetic energies of the incoming cosmic ray muons at various detector layers by quantifying the average energy loss in the target medium and analyzing the statistical properties of the deposited energy. The outcomes not only verify the consistency between the simulation and the reference data but also demonstrate the suitability of the GEANT4 simulations for the detailed studies of the energy deposition that are relevant to both the fundamental research and the applied muon imaging technologies. This study is organized as follows. Section 2 describes the methodology that is pursued for the computation of the energy deposition by the cosmic ray muons in the target volumes. Section 3 states the simulation properties in this study, while section 4 compares the simulation results with the literature for a list of materials that is composed of aluminum, copper, iron, lead, and uranium. Finally, the conclusions are drawn in section 5.

2 Methodology

In accordance with [6], the Bethe-Bloch equation is described as

$$< -\frac{dE}{dX} >= Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} \right) - \beta^2 - \frac{\delta}{2} \right] \tag{1}$$

where E is the total energy in MeV, $K = 4\pi N_A r_e^2 m_e c^2$ in MeV mol^{-1} cm^2, Z is the atomic number, A is the atomic weight in g/mol, $\beta = v/c$, $\gamma = 1/(1-\beta^2)^{\frac{1}{2}}$, z is the particle charge, m_e is the electron mass, $T_{\max}$ is the maximum kinetic energy which can be imparted to a free electron in a single collision, I is the mean excitation energy in eV, and δ is the density effect correction to the ionization energy loss. It should be carefully noted that X is defined as the mass thickness, i.e. the product of density and thickness, hence the units of $-dE/dX$ is in MeV g^{-1} cm^2 where $X = x\rho$ is the product of the medium density denoted by ρ in g cm^{-3} and the medium thickness indicated by x in cm.

In the present computational procedure, it is often necessary to determine the average energy loss per unit length as measured directly in a detector. This is typically accomplished by averaging the ratio between the difference in the kinetic energies of the cosmic ray muons and a given thickness of the associated material. The average energy loss per unit length can then be separated into contributions from the target material and any other intervening media such as air although the latter is generally negligible for most of the purposes. The following expression represents this approach:

$$< -\frac{dE}{dx} >_{\text{Total}} = < -\frac{dE}{dx} >_{\text{Target}} + \underbrace{< -\frac{dE}{dx} >_{\text{Air}}}_{\text{Negligible}} \approx < -\frac{dE}{dx} >_{\text{Target}} = \frac{1}{N} \sum_{i=1}^{N} -\frac{E_i^{\text{Bottom}} - E_i^{\text{Top}}}{\text{Thickness}} \tag{2}$$

For the comparison as well as the validation with the data from Grom et al., 2001 [8], Eq.(2) is divided by the density as follows

$$< -\frac{1}{\rho} \frac{dE}{dx} >= < -\frac{1}{\rho} \frac{dE}{dx} >_{\text{Target}} = < -\frac{dE}{dX} >_{\text{Target}} = \frac{1}{N} \sum_{i=1}^{N} -\frac{E_i^{\text{Bottom}} - E_i^{\text{Top}}}{\text{Thickness} \times \text{Density}} \tag{3}$$

The kinetic energies at top hodoscope and bottom hodoscope are recorded into a file throughout the simulation process, and the output file is post-processed by using Eq.(3).

3 Simulation scheme

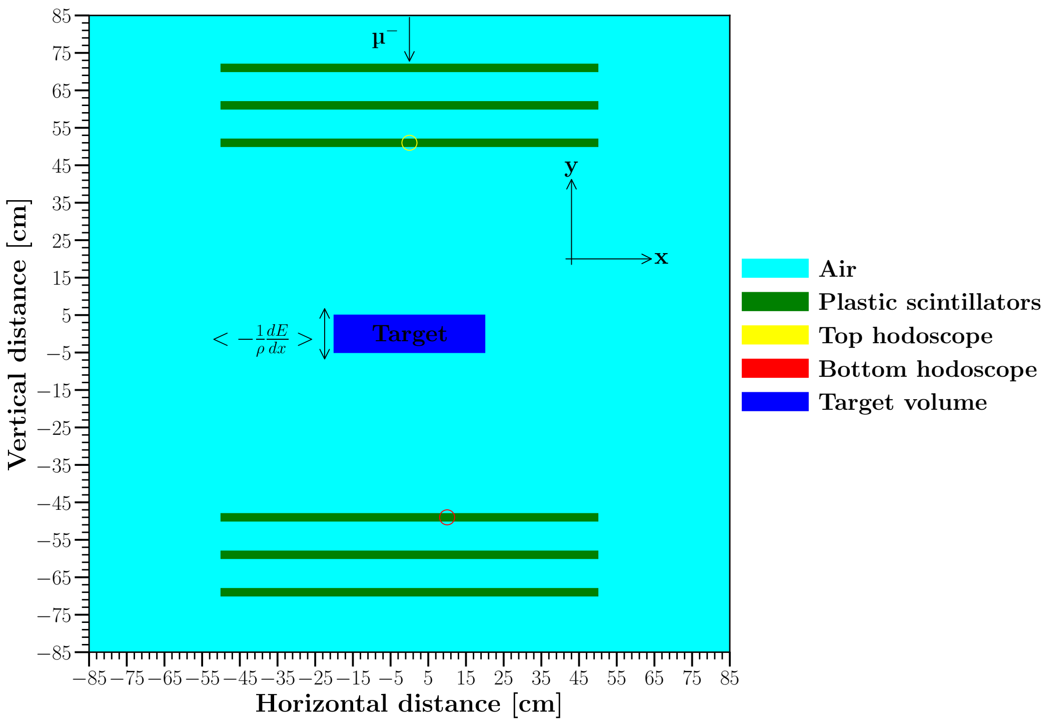

Figure 1: Simulation scheme to compute the energy deposition by utilizing a muon tomography hodoscope where the kinetic energy tracked at the third top layer denoted by E_i^{Top} is indicated by o, while the kinetic energy registered at the third top layer labeled as E_i^{Bottom} is pointed out by **o**.

The simulation scheme of the muon scattering tomography is illustrated in Fig. 1 where the top hodoscope and the bottom hodoscope consist of three plastic scintillators manufactured from polystyrene. The entering muons through the tomographic setup are tracked in the last top detector layer as well as the bottom detector layer in order to determine the energy loss by the cosmic ray muons in the corresponding materials. The difference between the kinetic energies at these detector sections means the energy loss by the incoming muons in the target volume of 40×10×40 cm^3. As described in Fig. 1, the dimensions of each the detector layer are $100 \times 0.4 \times 100$ cm^3, and the inter-layer spacing at each section is approximately 10 cm. Into the bargain, the gap between the top hodoscope and the bottom hodoscope is around 100 cm.

The simulation features in the present study are listed in Table 1. As afore-mentioned, the kinetic energies of the incoming muons on the detector layers fabricated from polystrene are initially tracked in order to calculate the mean energy loss as well as its corresponding standard deviations. A central mono-directional beam of constant energy that is generated at y=85 cm via G4ParticleGun is employed, and the generated muons are crossing in the vertically downward direction, i.e. from the top edge of the simulation box through the bottom edge as indicated with the black arrow in Fig. 1. A list of mono-energetic beams composed of 0.4, 1, 4, and 10 GeV is preferred for the sake of comparison. The reference physics list that is used in this study is FTFP_BERT, and the materials are defined in accordance with the G4/NIST database. The output file is compiled through G4Step, and the post-processing is maintained via a Python script where the difference between the kinetic energies is first calculated, then this subtraction is divided by the product of the thickness and the density. In the wake of this calculation, the corresponding ratio is averaged over the number of particles as shown in Eq. (3) by recalling that the number of particles in the current study is 10^5.

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Table 1: Simulation properties.

Particle	μ^-
Beam direction	Vertical
Momentum direction	(0, -1, 0)
Source geometry	Planar
Initial position (cm)	([-0.5, 0.5], 85, [-0.5, 0.5])
Number of particles	10^5
Energy (GeV)	0.4, 1, 4, and 10
Energy distribution	Constant
Material database	G4/NIST
Reference physics list	FTFP_BERT

4 Comparison and validation

To evaluate the accuracy of the simulation results, the mean stopping power values ($< -\frac{1}{\rho}\frac{dE}{dx} >$) obtained from the GEANT4 simulations are directly compared with the established reference data from Groom et al., 2001 [8]. Tables 2 and 3 present the simulated as well as literature values for a range of materials that include aluminum, copper, iron, lead, and uranium and also muon energies concerning 0.4, 1, 4, and 10 GeV, respectively.

Table 2: Simulated energy loss for a set of materials including aluminum, copper, iron, lead, and uranium by using GEANT4.

	$< -\frac{1}{\rho}\frac{dE}{dx} > \pm\delta(-\frac{1}{\rho}\frac{dE}{dx})$ [MeV cm^2 g^{-1}]			
Material	0.4 GeV	1 GeV	4 GeV	10 GeV
Aluminum	1.659±0.171	1.772±0.374	2.011±1.299	2.151±2.928
Copper	1.421±0.084	1.528±0.188	1.774±0.729	1.942±1.729
Iron	1.471±0.091	1.580±0.207	1.821±0.759	1.997±1.870
Lead	1.150±0.069	1.260±0.155	1.506±0.659	1.686±1.497
Uranium	1.112±0.052	1.205±0.114	1.446±0.506	1.641±1.299

The simulation outcomes closely match with the literature values across all materials and energies. For example, the simulated stopping power for 1-GeV muons in aluminum is 1.772 ± 0.374 MeV cm^2 g^{-1}, which is in excellent agreement with the reference value of 1.745 MeV cm^2 g^{-1}. In contrast, the dissimilarities for the other data points remain within a few percent, which is well tolerable within the statistical uncertainties of the Monte Carlo simulations.

Table 3: Energy loss for a set of materials consisting of aluminum, copper, iron, lead, and uranium as stated in Groom et al, 2001 [8].

	Groom et al. (2001) [8] [MeV cm^2 g^{-1}]			
Material	0.4 GeV	1 GeV	4 GeV	10 GeV
Aluminum	1.630	1.745	1.971	2.117
Copper	1.419	1.533	1.766	1.927
Iron	1.467	1.582	1.816	1.975
Lead	1.152	1.272	1.508	1.692
Uranium	1.109	1.225	1.455	1.639

This agreement confirms that the GEANT4 framework accurately reproduces the mean energy deposition of muons in various materials. Thus, the simulation approach is validated for use in practical applications involving muon energy loss calculations and detector studies in the muon scattering tomography.

5 Conclusion

All in all, the mean energy deposited within the target volume is determined by dint of the GEANT4 simulations, and the simulation outcomes are compared with those of Groom et al., 2001. It is shown that a GEANT4 framework might be an effective tool for the energy loss calculations.

6 Copyright Notice

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