London Journal of PHYSICS
Volume 2 No 2

Classic Doppler Effect: An Extended Formula

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October 27, 2025

Abstract

Following a revisited approach to the classic Doppler effect, we will propose an extended Doppler effect formula, available in all cases of speeds and directions of any mechanic wave emitter and receiver at a uniform speed. In a first step, we will apply a Doppler simulator to gradually more complex cases and compare these results with those established by geometric construction. In a second step, we will focus on limit cases and compare the Doppler ratios provided by the accepted formula and the extended one established in this study.

1 Doppler simulator

A numeric simulator enables us to generate concentric periodic waves typically affected by the Doppler effect, and then to place a receiver anywhere nearby the emitter in order to establish the Doppler ratio.

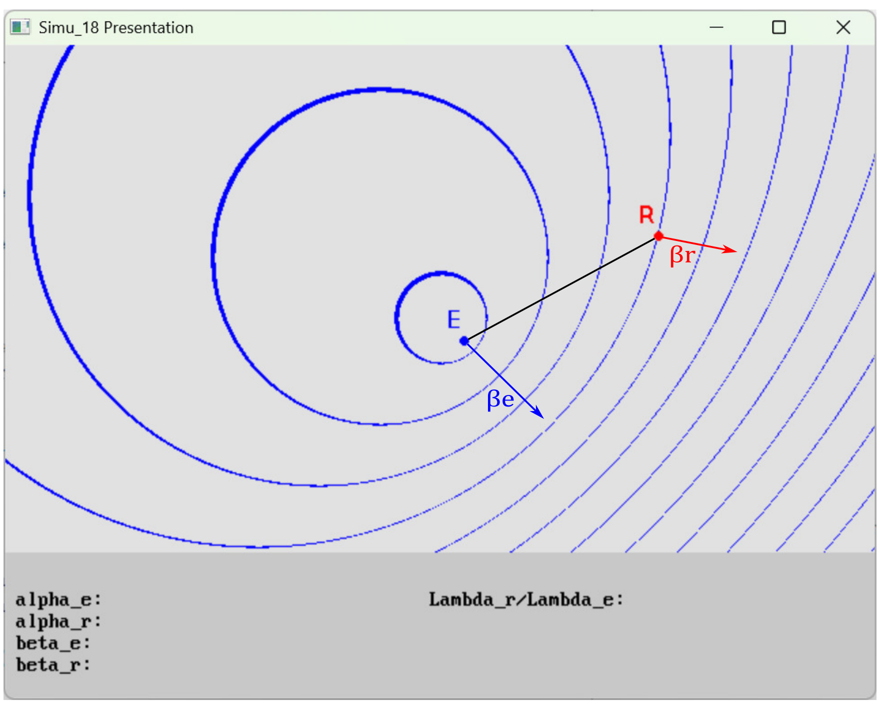


Figure 1: Screenshot of the simulator.

For the whole study, we will use the following notations:

* c $(m.s^{-1})$: speed of a mechanic wave

* $\lambda_e = \frac{c}{f_e}$ (m) : wavelength (resp. frequency f_e in Hz) of the emitter

* $\lambda_r = \frac{c}{f_r}$ (m) : wavelength (resp. frequency f_r in Hz) measured by the receiver

* $\beta_e = v_e/c$: speed of the emitter normalized to the speed of the wave

* $\beta_r = v_r/c$: speed of the receiver normalized to the speed of the wave

* α_e (rad) : angle between the speed of the emitter and the axis "Emitter-Receiver" at the time of emission

* α_r (rad) : angle between the speed of the receiver and the axis "Emitter-Receiver" at the time of emission

By definition, the Doppler ratio or Doppler effect value is:

* In terms of wavelength:

$$\frac{\lambda_r}{\lambda_e}$$

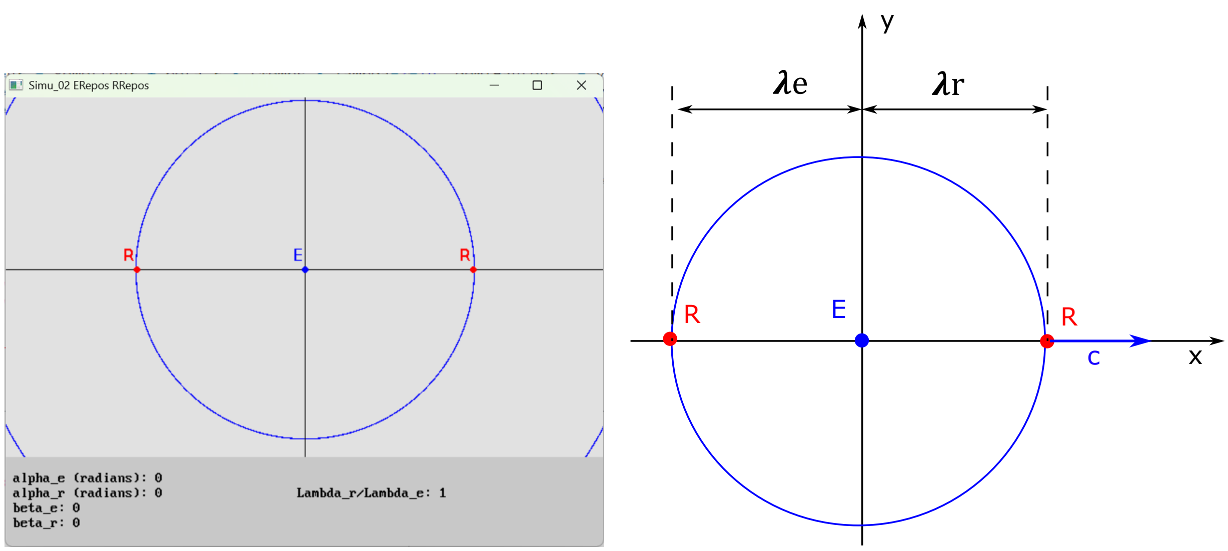
* In terms of frequency:

$$\frac{f_r}{f_e}$$

2 Case review of the Doppler effect

The first step consists in building a review of the Doppler ratios for increasingly complex cases in terms of position and movement of an emitter and a receiver.

2.1 The emitter and the receiver are static



(a) Simulation. (b) Schematic.

Figure 2: The emitter and the receiver are static.

The received and measured frequency trivially equals to the emitted frequency:

$$f_r = f_e \tag{1}$$

That is, for the wavelengths:

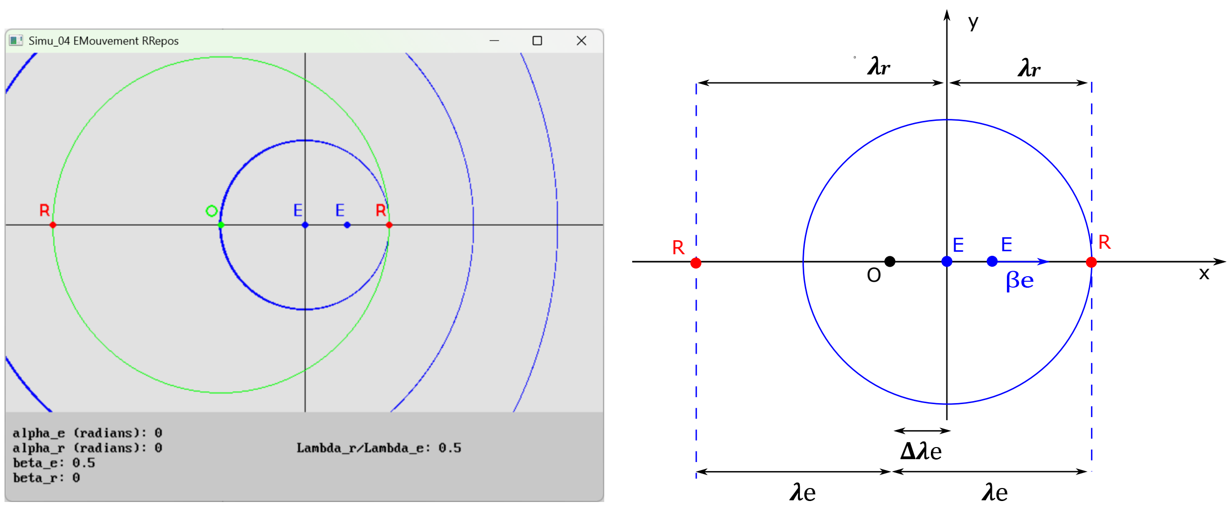
$$\lambda_r = \lambda_e \tag{2}$$

According to the simulator and the formula previously established, we get:

Table 1: Table of results when the emitter and the receiver are static

Input Data	Simulator	Formula
$\alpha_e = 0$		
$\alpha_r = 0$	$\frac{\lambda_r}{\lambda_e} = 1$	$\frac{\lambda_r}{\lambda_e} = 1$
$\beta_e = 0$		
$\beta_r = 0$		

2.2 The emitter moves, the receiver is static



(a) Simulation. (b) Schematic.

Figure 3: The emitter moves, the receiver is static.

We have:

$$\Delta\lambda_e = \beta_e.\lambda_e \tag{3}$$

For the receiver at the front:

$$\lambda_r = \lambda_e.(1 - \beta_e) \tag{4}$$

For the receiver at the back:

$$\lambda_r = \lambda_e.(1 + \beta_e) \tag{5}$$

According to the simulator and the formula previously established, we get for the receiver at the front:

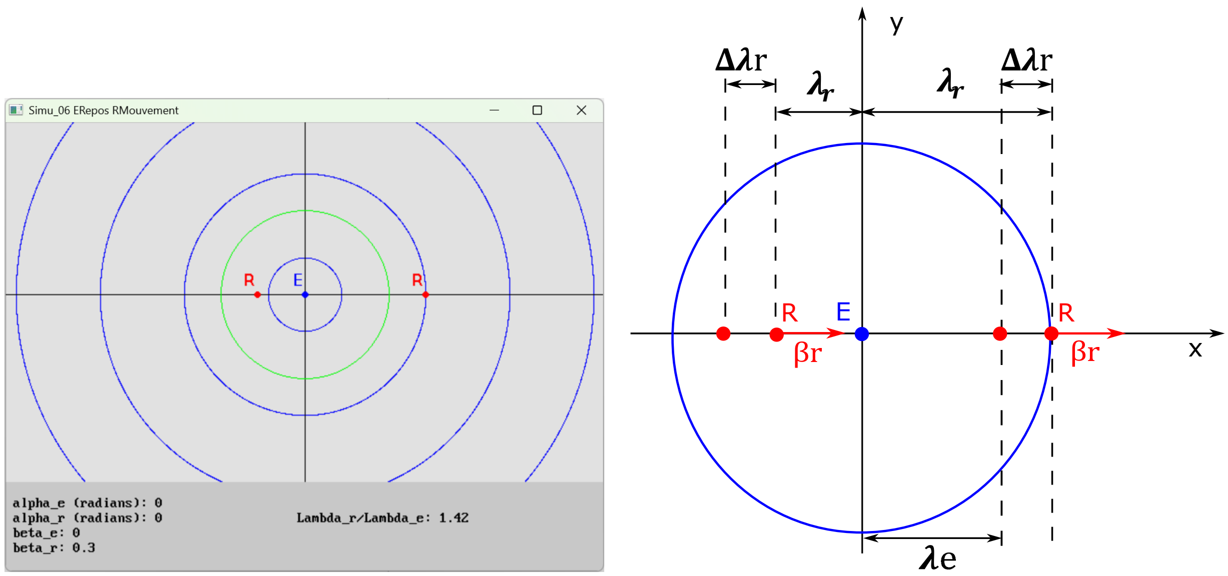
2.3 The emitter is static, the receiver moves

We have:

 DOI 10.69710/ljp.v2i2.15761

Table 2: Table of results when the emitter moves, the receiver is static

Input Data	Simulator	Formula
$\alpha_e = 0$ $\alpha_r = 0$ $\beta_e = 0.5$ $\beta_r = 0$	$\frac{\lambda_r}{\lambda_e} = 0.5$	$\frac{\lambda_r}{\lambda_e} = 0.5$



(a) Simulation. (b) Schematic.

Figure 4: The emitter is static, the receiver moves.

$$\Delta\lambda_r = \beta_r.\lambda_r \tag{6}$$

The wavelength measured by the receiver at the front equals to:

$$\lambda_r = \frac{\lambda_e}{1 - \beta_r} \tag{7}$$

The wavelength measured by the receiver at the back equals to:

$$\lambda_r = \frac{\lambda_e}{1 + \beta_r} \tag{8}$$

According to the simulator and the formula previously established, we get for the receiver at the front:

2.4 The emitter and the receiver move on the same axis

By considering the movement of both the receiver and the emitter, the wavelength measured by the receiver at the front equals to:

$$\lambda_r = \lambda_e.\frac{1 - \beta_e}{1 - \beta_r} \tag{9}$$

The wavelength measured by the receiver at the back equals to:

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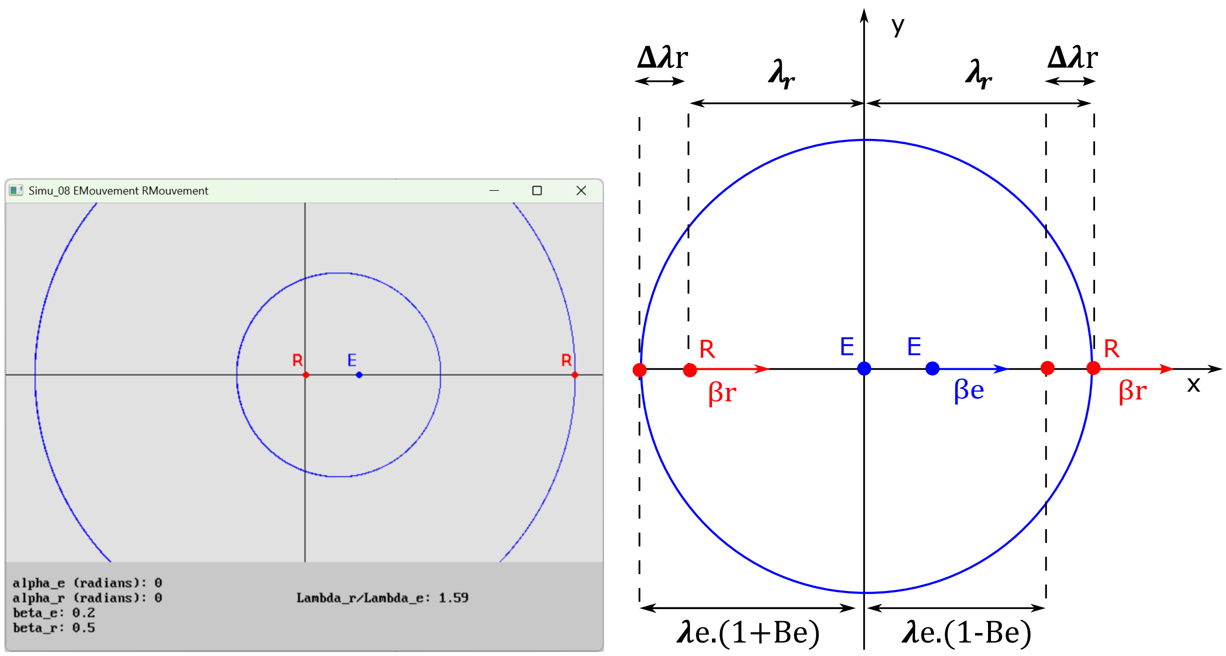
Table 3: Table of results when the emitter is static, the receiver moves

Input Data	Simulator	Formula
$\alpha_e = 0$		
$\alpha_r = 0$	$\frac{\lambda_r}{\lambda_e} = 1.42$	$\frac{\lambda_r}{\lambda_e} = 1.43$
$\beta_e = 0$		
$\beta_r = 0.3$		



(a) Simulation. (b) Schematic.

Figure 5: The emitter and the receiver move on the same axis.

$$\lambda_r = \lambda_e.\frac{1+\beta_e}{1+\beta_r} \tag{10}$$

Note:

If $\beta_e = \beta_r$, then $\lambda_r = \lambda_e$. The relative Doppler effect is therefore null.

According to the simulator and the formula previously established, we get for the receiver at the front:

2.5 The emitter moves, the receiver is static on another axis

Let us call the angle between the direction of the emitter and the axis "emitter-receiver" at the emission point when the speed of the emitter tends to zero : α_0

We have:

$$\lambda_{\alpha_e} = RM - EM \tag{11}$$

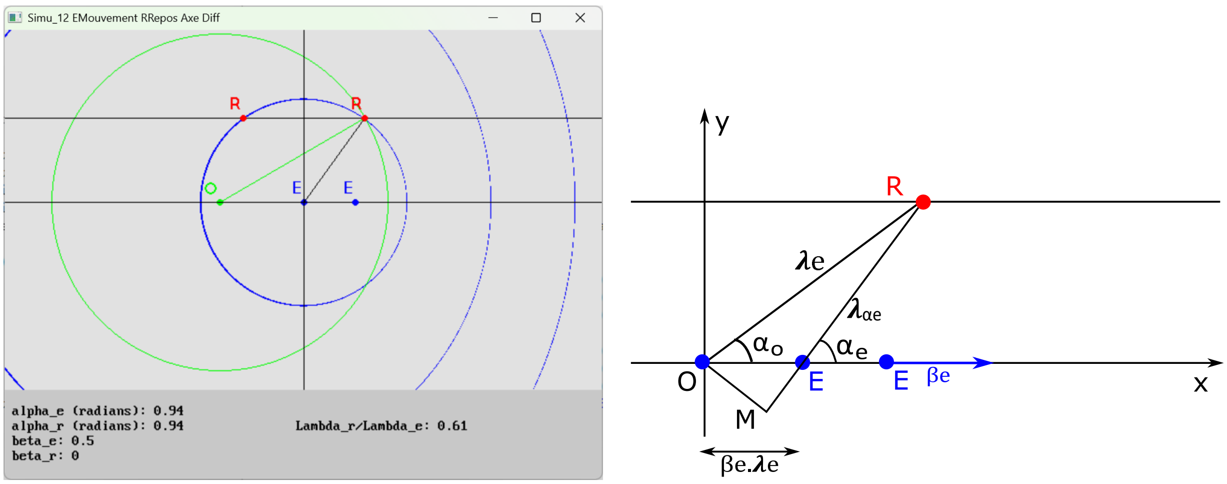
With:

$$EM = \lambda_e.\beta_e.\cos\alpha_e \tag{12}$$

 DOI 10.69710/ljp.v2i2.15761

Table 4: Table of results when the emitter and the receiver move on the same axis

Input Data	Simulator	Formula
$\alpha_e = 0$ $\alpha_r = 0$ $\beta_e = 0.2$ $\beta_r = 0.5$	$\frac{\lambda_r}{\lambda_e} = 1.59$	$\frac{\lambda_r}{\lambda_e} = 1.6$



(a) Simulation. (b) Schematic.

Figure 6: The emitter moves, the receiver is static on another axis.

And:

$$RM = \lambda_e.\sqrt{1 - \beta_e^2.\sin^2\alpha_e} \tag{13}$$

This leads to:

$$\lambda_{\alpha_e} = \lambda_e.(\sqrt{1 - \beta_e^2.\sin^2\alpha_e} - \beta_e.\cos\alpha_e) \tag{14}$$

Besides, we have in this case:

$$\lambda_{\alpha_e} = \lambda_r \tag{15}$$

Thus, in this case:

$$\frac{\lambda_r}{\lambda_e} = \sqrt{1 - \beta_e^2.\sin^2\alpha_e} - \beta_e.\cos\alpha_e \tag{16}$$

According to the simulator and the formula previously established, we get for the receiver at the front:

2.6 The receiver moves, the emitter is static on another axis

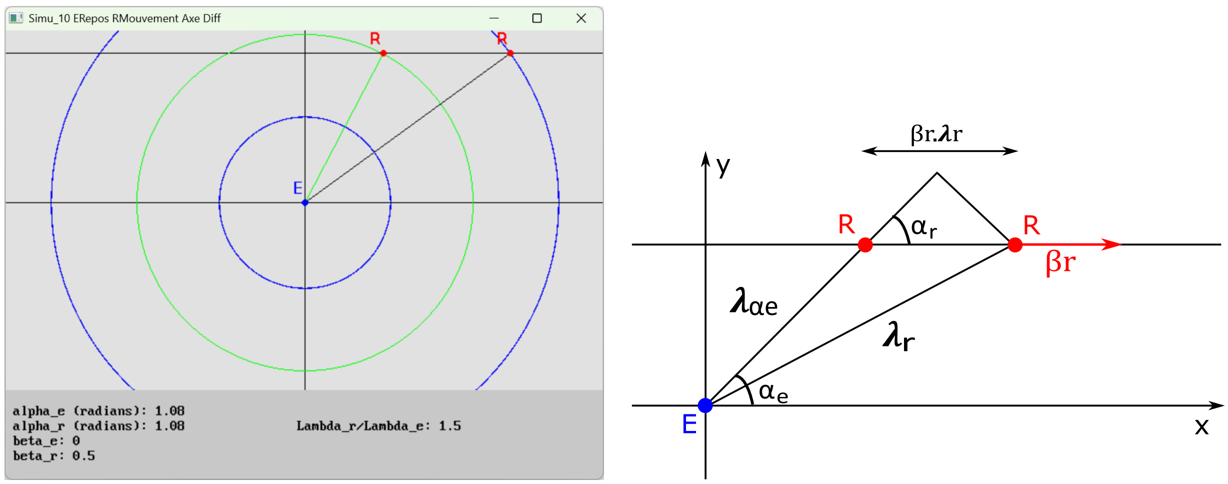
As $\alpha_r = \alpha_e$, we have:

$$\lambda_r^2 = (\lambda_{\alpha_e} + \beta_r.\lambda_r.\cos\alpha_e)^2 + \beta_r^2.\lambda_r^2.\sin^2\alpha_e \tag{17}$$

$$\lambda_r^2.(1 - \beta_r^2) - 2\beta_r.\lambda_r.\lambda_{\alpha_e}.\cos\alpha_e - \lambda_{\alpha_e}^2 = 0 \tag{18}$$

Table 5: Table of results when the emitter moves, the receiver is static on another axis

Input Data	Simulator	Formula
$\alpha_e = 0.94$		
$\alpha_r = 0.94$	$\frac{\lambda_r}{\lambda_e} = 0.61$	$\frac{\lambda_r}{\lambda_e} = 0.62$
$\beta_e = 0.5$		
$\beta_r = 0$		



(a) Simulation. (b) Schematic.

Figure 7: The receiver moves, the emitter is static on another axis.

Which leads to the remarkable expression:

$$\lambda_r = \frac{\lambda_{\alpha_e}}{\sqrt{1 - \beta_r^2 . \sin^2 \alpha_e} - \beta_r . \cos \alpha_e} \tag{19}$$

Besides, we have in this case:

$$\lambda_{\alpha_e} = \lambda_e \tag{20}$$

Thus, in this case:

$$\frac{\lambda_r}{\lambda_e} = \frac{1}{\sqrt{1 - \beta_r^2 . \sin^2 \alpha_e} - \beta_r . \cos \alpha_e} \tag{21}$$

According to the simulator and the formula previously established, we get:

Table 6: Table of results when the receiver moves, the emitter is static on another axis

Input Data	Simulator	Formula
$\alpha_e = 1.08$		
$\alpha_r = 1.08$	$\frac{\lambda_r}{\lambda_e} = 1.50$	$\frac{\lambda_r}{\lambda_e} = 1.51$
$\beta_e = 0$		
$\beta_r = 0.5$		

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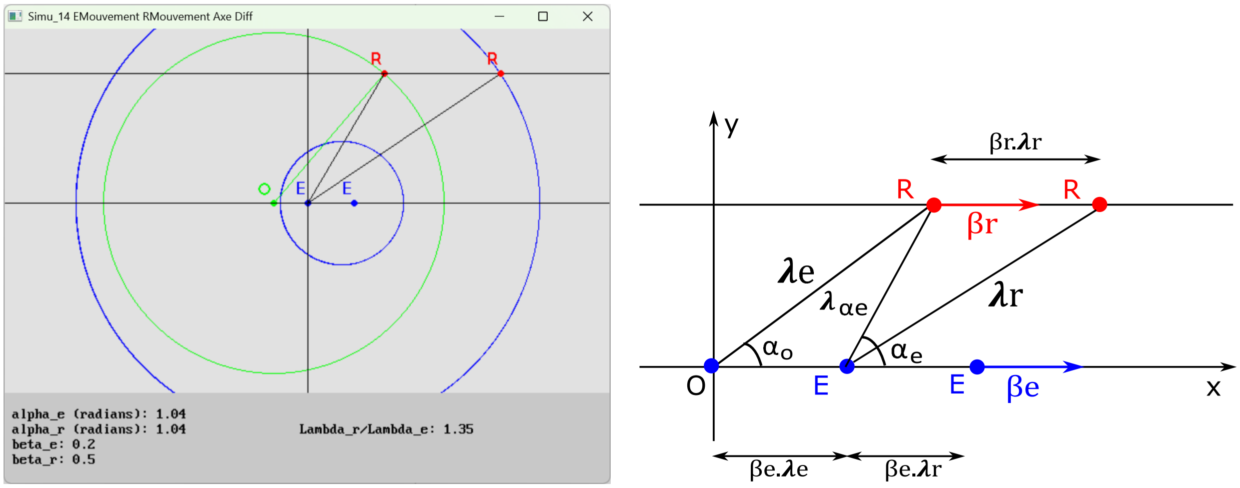
2.7 The receiver and the emitter move on two different axis



(a) Simulation. (b) Schematic.

Figure 8: The receiver and the emitter move on two different axis.

We assume that the emitter and the receiver move in a colinear way.

Combining (14) and (19) leads to the following expression for the Doppler effect:

$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1-\beta_e^2.\sin^2\alpha_e} - \beta_e.\cos\alpha_e}{\sqrt{1-\beta_r^2.\sin^2\alpha_e} - \beta_r.\cos\alpha_e} \tag{22}$$

According to the simulator and the formula previously established, we get:

Table 7: Table of results when the receiver and the emitter move on two different axis

Input Data	Simulator	Formula
$\alpha_e = 1.04$ $\alpha_r = 1.04$ $\beta_e = 0.2$ $\beta_r = 0.5$	$\frac{\lambda_r}{\lambda_e} = 1.35$	$\frac{\lambda_r}{\lambda_e} = 1.36$

2.8 The receiver and the emitter move in a non-collinear way

Let us consider the angle α_r in the formula (17) instead of α_e, we get then:

$$\lambda_r^2 = (\lambda_{\alpha_e} + \beta_r.\lambda_r.\cos\alpha_r)^2 + \beta_r^2.\lambda_r^2.\sin^2\alpha_r \tag{23}$$

That is:

$$\lambda_r = \frac{\lambda_{\alpha_e}}{\sqrt{1-\beta_r^2.\sin^2\alpha_r} - \beta_r.\cos\alpha_r} \tag{24}$$

Thus, the extended Doppler formula becomes:

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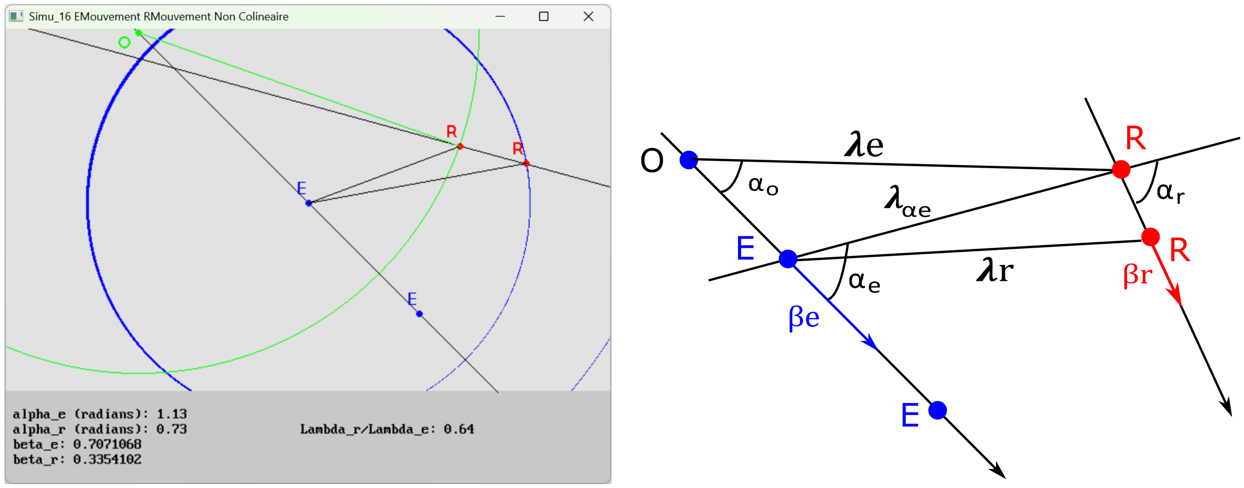

(a) Simulation. (b) Schematic.

Figure 9: The receiver and the emitter move in a non-collinear way.

$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1 - \beta_e^2.\sin^2\alpha_e} - \beta_e.\cos\alpha_e}{\sqrt{1 - \beta_r^2.\sin^2\alpha_r} - \beta_r.\cos\alpha_r} \tag{25}$$

According to the simulator and the formula previously established, we get:

Table 8: Table of results when the receiver and the emitter move in a non-collinear way

Input Data	Simulator	Formula
$\alpha_e = 1.13$ $\alpha_r = 0.73$ $\beta_e = 0.707$ $\beta_r = 0.335$	$\frac{\lambda_r}{\lambda_e} = 0.64$	$\frac{\lambda_r}{\lambda_e} = 0.64$
$\alpha_e = 1.13$ $\alpha_r = 3.87$ $\beta_e = 0.707$ $\beta_r = 0.335$	$\frac{\lambda_r}{\lambda_e} = 0.38$	$\frac{\lambda_r}{\lambda_e} = 0.38$
$\alpha_e = 1.13$ $\alpha_r = 3.105$ $\beta_e = 0.707$ $\beta_r = 0.335$	$\frac{\lambda_r}{\lambda_e} = 0.35$	$\frac{\lambda_r}{\lambda_e} = 0.35$
$\alpha_e = 1.13$ $\alpha_r = 6.245$ $\beta_e = 0.707$ $\beta_r = 0.335$	$\frac{\lambda_r}{\lambda_e} = 0.69$	$\frac{\lambda_r}{\lambda_e} = 0.70$

3 Limit cases

In the litteracy dedicated to the study of waves and the Doppler effect in particular, it is possible to find an alternative formula [1] which is in our sense uncomplete:

$$\frac{\lambda_r}{\lambda_e} = \frac{1 - \beta_e . \cos \alpha_e}{1 - \beta_r . \cos \alpha_r} \tag{26}$$

This is what we will see in the next subsections by pushing the Doppler effect experiment to limit cases. At the same time, we will see that the extended Doppler formula (25) provides more relevant results.

3.1 The receiver moves at the speed of the wave

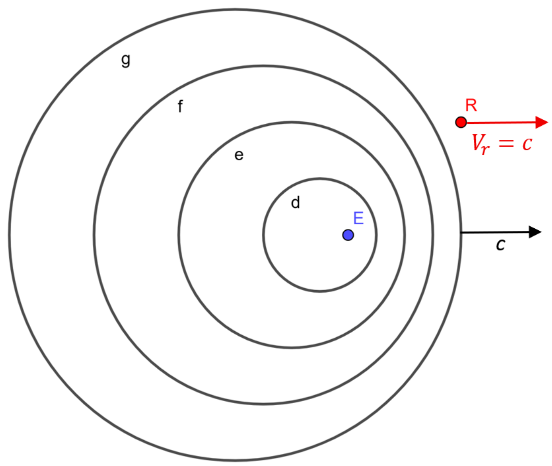

Figure 10: The receiver moves at the speed of the wave.

If $\beta_r = 1$, then:

$$\sqrt{1 - \beta_r^2 . \sin^2 \alpha_r} = \pm \cos \alpha_r \tag{27}$$

If $\frac{\pi}{2} < \alpha_r < \frac{3.\pi}{2}$, then according to formula (25):

$$\frac{\lambda_r}{\lambda_e} = -\frac{\sqrt{1 - \beta_e^2 . \sin^2 \alpha_e} - \beta_e . \cos \alpha_e}{2. \cos \alpha_r} \tag{28}$$

If $-\frac{\pi}{2} < \alpha_r < \frac{\pi}{2}$, then according to formula (25):

$$\frac{\lambda_r}{\lambda_e} \to \infty \tag{29}$$

Which means that the wave is not able to reach the receiver as far as the latter moves away at the speed of the former.

Let us note that, according to the accepted formula (26), the Doppler ratio would be in this case:

$$\frac{\lambda_r}{\lambda_e} = \frac{1 - \beta_e . \cos \alpha_e}{1 - \cos \alpha_r} \tag{30}$$

This formula involves that the Doppler ratio would tend to infinity in the only case where $\alpha_r = 0$. Yet, an experimental simulation shows that the emitted wave is not able to reach the receiver as far as the latter is located in front of the former, not only in the case where $\alpha_r = 0$ but whatever α_r between $-\pi/2$ and $+\pi/2$.

In the present case, the new formula represents experience more accurately than the accepted formula.

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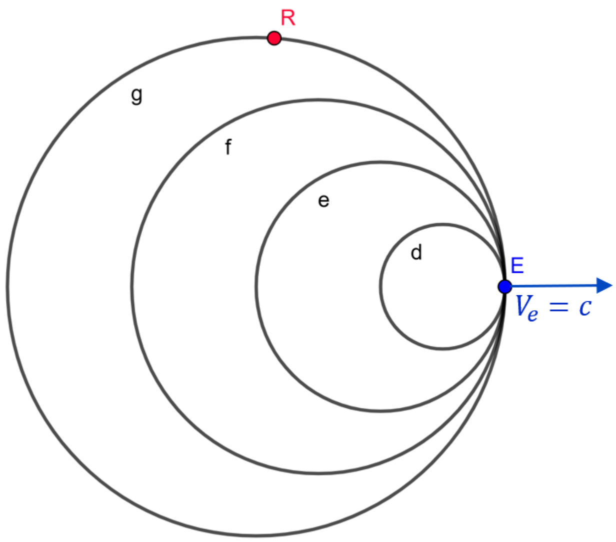

Figure 11:
The emitter moves at the speed of the wave - The receiver is static.

3.2 The emitter moves at the speed of the wave - The receiver is static

If $\beta_e = 1$, then:

$$\sqrt{1 - \beta_e^2 . \sin^2 \alpha_e} = \pm \cos \alpha_e \tag{31}$$

If moreover $\beta_r = 0$, we have according to formula (25):

$$\frac{\lambda_r}{\lambda_e} = \pm \cos \alpha_e - \cos \alpha_e \tag{32}$$

If $-\frac{\pi}{2} < \alpha_e < \frac{\pi}{2}$, then according to formula (25):

$$\frac{\lambda_r}{\lambda_e} = 0 \tag{33}$$

If $\frac{\pi}{2} < \alpha_e < \frac{3.\pi}{2}$, then according to formula (25):

$$\frac{\lambda_r}{\lambda_e} = -2. \cos \alpha_e \tag{34}$$

Let us note that, according the accepted formula (26) given by [1] and [2], we have:

$$\frac{\lambda_r}{\lambda_e} = 1 - \beta_e . \cos \alpha_e \tag{35}$$

That is, if $\beta_e = 1$:

$$\frac{\lambda_r}{\lambda_e} = 1 - \cos \alpha_e \tag{36}$$

Let us draw the two functions we want to compare:

$$f(\alpha_e) = 1 - \beta_e . \cos \alpha_e \tag{37}$$

$$g(\alpha_e) = \sqrt{1 - \beta_e^2 . \sin^2 \alpha_e} - \beta_e . \cos \alpha_e \tag{38}$$

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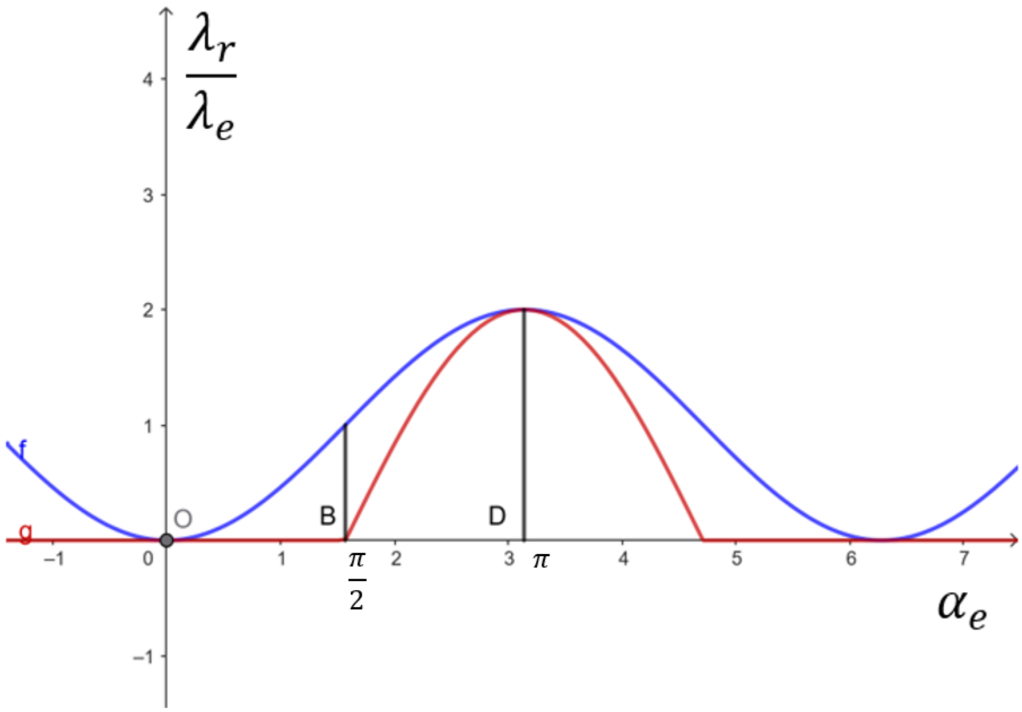

Figure 12:
Graphic representation of the two functions when $\beta_e = 1$

For $\beta_e = 1$, we can see that:

* If $\alpha_e = 0$ or $\alpha_e = \pi$: The results are similar and correct for both formulas

* If $0 < \alpha_e < +\frac{\pi}{2}$: The accepted formula (26) provides a result which doesn't match with the experiment since the receiver's position has to coincide with the emitter's position in this case. Therefore, the Doppler ratio is null, which is precisely the value provided by the extended Doppler formula (25).

Still in this case, the new formula represents experience more accurately than the accepted formula.

Nevertheless, there is a convergence between the results provided by the two formulas when β_e tends to zero.

If we consider for example the least favorable case where $\alpha_e = \frac{\pi}{2}$, we have the following results for the Doppler ratio:

Emitter's speed	Classic formula	Extended formula
β_e= 1	$\frac{\lambda_r}{\lambda_e} = 1$	$\frac{\lambda_r}{\lambda_e} = 0$
β_e= 0.5	$\frac{\lambda_r}{\lambda_e} = 1$	$\frac{\lambda_r}{\lambda_e} \approx 0.86$
β_e= 0.1	$\frac{\lambda_r}{\lambda_e} = 1$	$\frac{\lambda_r}{\lambda_e} \approx 0.995$

4 Conclusion

The Doppler effect extended formula established in this study enables to estimate the Doppler ratio in all cases of speeds and directions of a mechanic wave emitter and receiver when they move at a uniform speed:

$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1 - \beta_e^2 . \sin^2 \alpha_e} - \beta_e . \cos \alpha_e}{\sqrt{1 - \beta_r^2 . \sin^2 \alpha_r} - \beta_r . \cos \alpha_r}$$

 DOI 10.69710/ljp.v2i2.15761

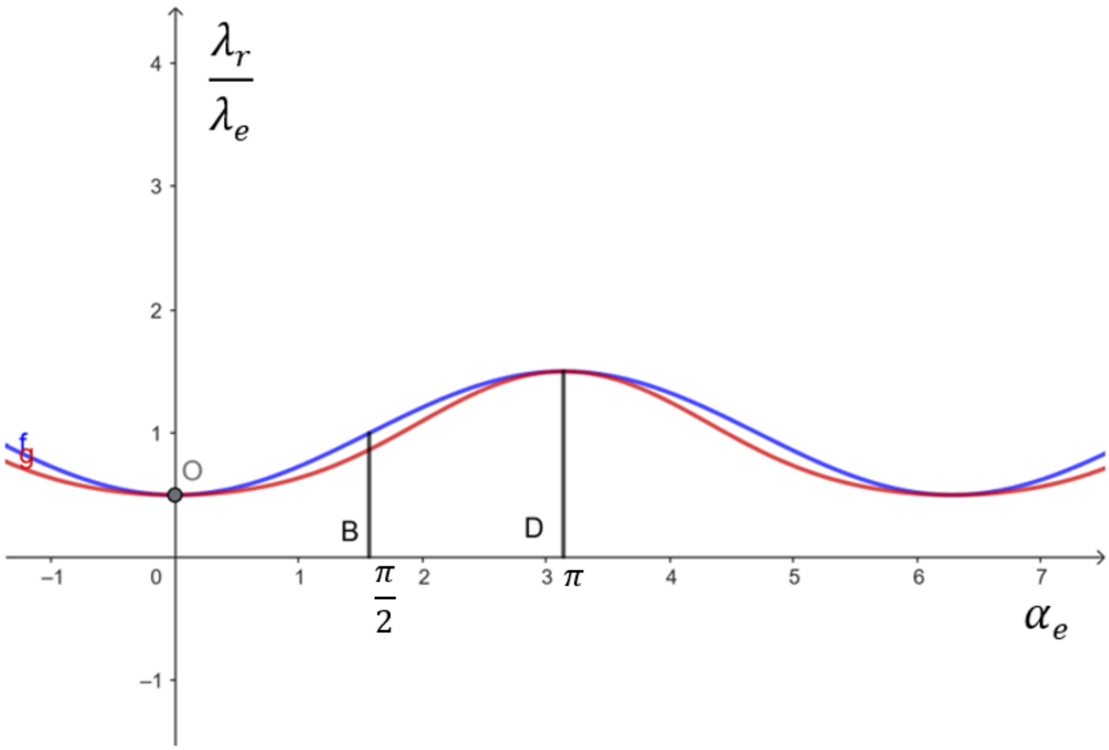

Figure 13:
Graphic representation of the two functions when $\beta_e = 0.5$

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References

[1] André Saint-Amand - Physique des ondes - 2008 - Presses Universitaires du Québec

[2] Matthew Schwarz - The Doppler Effect - Lecture 21 - Harvard Scholars
https://schwartz.scholars.harvard.edu/file_url/145