From Classic to Relativistic Doppler Effect

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Abstract

Based on an innovative understanding of special relativity combined to the Doppler effect extended formula which was established in our former study [1], we will expose an alternative formula for the Doppler effect in the relativistic case, and show how it can be used in modern astrophysics.

1 The classic Doppler effect extended formula

The Doppler effect extended formula already established [1] enables to estimate the Doppler ratio in all cases of speeds and directions of a mechanic wave emitter and receiver when they move at a uniform speed:

$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1 - \beta_e^2 . \sin^2 \alpha_e} - \beta_e . \cos \alpha_e}{\sqrt{1 - \beta_r^2 . \sin^2 \alpha_r} - \beta_r . \cos \alpha_r}$$

With:

* $c\ (m.s^{-1})$: speed of a mechanic wave

* $\lambda_e = c/f_e\ (m)$: wavelength (resp. frequency f_e in Hz) of the emitter

* $\lambda_r = c/f_r\ (m)$: wavelength (resp. frequency f_r in Hz) measured by the receiver

* $\beta_e = V_e/c$: speed of the emitter normalized to the speed of the wave

* $\beta_r = V_r/c$: speed of the receiver normalized to the speed of the wave

* $\alpha_e\ (rad)$: angle between the speed of the emitter and the axis "Emitter-Receiver" at the time of emission

* $\alpha_r\ (rad)$: angle between the speed of the receiver and the axis "Emitter-Receiver" at the time of emission

By definition, the Doppler ratio or Doppler effect value is:

* In terms of wavelength:

$$\frac{\lambda_r}{\lambda_e}$$

* In terms of frequency:

$$\frac{f_r}{f_e}$$

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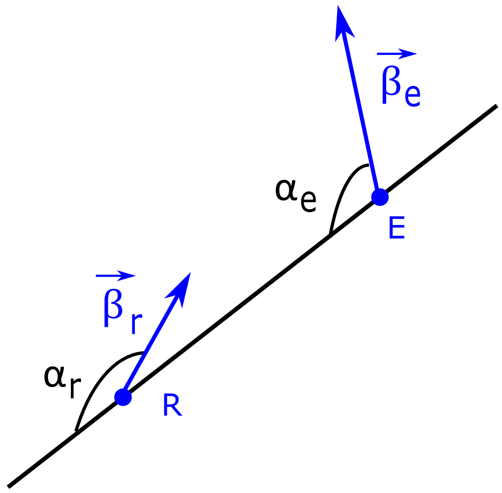

Figure 1: Graphic representation for the extended formula

2 Property of an electromagnetic emitter/receiver

For the rest of our study, we will assume that an electromagnetic wave emitter or receiver is influenced by movement according to the Lorentz factor g. If β is the speed of the emitter or receiver normalized to the speed of an electromagnetic wave, that is to the speed of light in vacuum, then:

$$g = \sqrt{1 - \beta^2}$$

If we focus first on an electromagnetic wave emitter then we assume, like Albert Einstein did in his early works of 1907 [2], that its frequency is slowed down according to the Lorentz factor g_e compared to its fundamental frequency f_e, when it moves at the speed β_e:

$$f_{\beta_e} = g_e.f_e \tag{1}$$

$$g_e = \sqrt{1 - \beta_e^2} \tag{2}$$

Therefore, the wavelength of a moving emitter equals to:

$$\lambda_{\beta_e} = \frac{\lambda_e}{g_e} \tag{3}$$

If we focus then on an electromagnetic wave receiver, we assume the same way that its measured frequency is slowed down according to the Lorentz factor g_r compared to its fundamental frequency f_r, when it moves at the speed β_r:

$$f_{\beta_r} = g_r.f_r \tag{4}$$

$$g_r = \sqrt{1 - \beta_r^2} \tag{5}$$

Therefore, the measured wavelength of a moving receiver equals to:

$$\lambda_{\beta_r} = \frac{\lambda_r}{g_r} \tag{6}$$

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3 The relativistic Doppler effect formula

If we take into account the influence of the movement on the emitter and the receiver frequency properties, it is then possible to establish from the first formula of our study the following relativistic Doppler effect formula:

$$\frac{\lambda_{\beta_r}}{\lambda_{\beta_e}} = \frac{\sqrt{1-\beta_e^2.\sin^2\alpha_e} - \beta_e.\cos\alpha_e}{\sqrt{1-\beta_r^2.\sin^2\alpha_r} - \beta_r.\cos\alpha_r} \tag{7}$$

That is:

$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1-\beta_r^2}}{\sqrt{1-\beta_e^2}}.\frac{\sqrt{1-\beta_e^2.\sin^2\alpha_e} - \beta_e.\cos\alpha_e}{\sqrt{1-\beta_r^2.\sin^2\alpha_r} - \beta_r.\cos\alpha_r} \tag{8}$$

4 Classic and alternative formula in the longitudinal case

4.1 Reminder of the classic formula

The classic formula for the relativistic Doppler effect [3] was established by applying the Lorentz transformations to the four-momentum of a photon. Let us focus on the time component of the momentum expressed in the proper reference frame of the photon in a first time, and in a second time in the reference frame of an observer moving at the relative speed β:

$$E' = \frac{E}{\sqrt{1-\beta^2}}.(1-\beta.cos\phi)$$

Where ϕ is the angle between the relative speed β and the axis "Emitter-Receiver"

The equivalence $E = h.f$ and $E' = h.f'$ where h is the Planck constant ; f and f' are the photon's frequencies in the two respectives reference frames ; leads to the following Doppler expression:

$$f' = f.\frac{1-\beta.cos\phi}{\sqrt{1-\beta^2}}$$

Using the notations of this study enables us to write:

$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1-\beta^2}}{1-\beta.cos\phi} \tag{9}$$

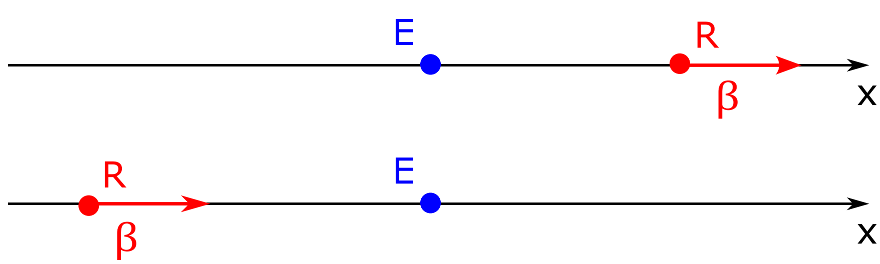

Figure 2:
The emitter and the receiver are in relative motion on the same axis

In the case where the emitter and the receiver move away from each other ($\phi = 0$), the relativistic formula leads to:

$$\lambda_r = \lambda_e.\sqrt{\frac{1+\beta}{1-\beta}} \tag{10}$$

In the case where the emitter and the receiver approach to each other ($\phi = \pi$), the relativistic formula leads to:

$$\lambda_r = \lambda_e.\sqrt{\frac{1-\beta}{1+\beta}} \tag{11}$$

4.2 Alternative formula

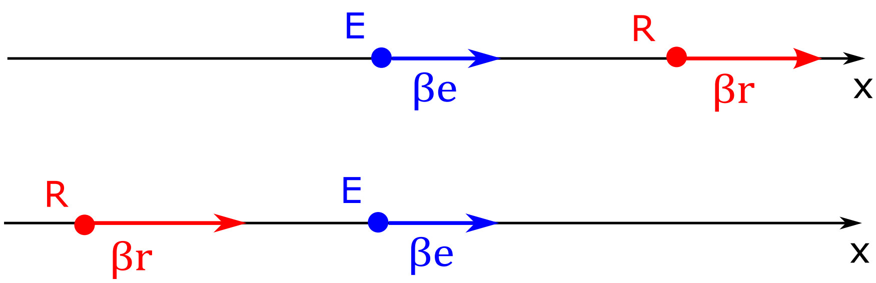

Figure 3:
The emitter and the receiver move on the same axis

For the receiver to the front of the emitter ($\alpha_e = 0, \alpha_r = 0$), we get the following formula according to equation (8):

$$\frac{\lambda_r}{\lambda_e} = \sqrt{\frac{(1+\beta_r).(1-\beta_e)}{(1+\beta_e).(1-\beta_r)}} \tag{12}$$

$$\frac{\lambda_r}{\lambda_e} = \sqrt{\frac{1+(\beta_r-\beta_e)-\beta_r.\beta_e}{1-(\beta_r-\beta_e)-\beta_r.\beta_e}} \tag{13}$$

For the receiver to the back of the emitter ($\alpha_e = \pi, \alpha_r = \pi$), we get the following formula according to equation (8):

$$\frac{\lambda_r}{\lambda_e} = \sqrt{\frac{(1-\beta_r).(1+\beta_e)}{(1-\beta_e).(1+\beta_r)}} \tag{14}$$

$$\frac{\lambda_r}{\lambda_e} = \sqrt{\frac{1-(\beta_r-\beta_e)-\beta_r.\beta_e}{1+(\beta_r-\beta_e)-\beta_r.\beta_e}} \tag{15}$$

4.3 Relativistic speed composition

Equation (13) can also be written::

$$\frac{\lambda_r}{\lambda_e} = \sqrt{\frac{1+\frac{\beta_r-\beta_e}{1-\beta_r.\beta_e}}{1-\frac{\beta_r-\beta_e}{1-\beta_r.\beta_e}}} \tag{16}$$

Equation (15) can also be written::

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$$\frac{\lambda_r}{\lambda_e} = \sqrt{\frac{1 - \frac{\beta_r - \beta_e}{1 - \beta_r.\beta_e}}{1 + \frac{\beta_r - \beta_e}{1 - \beta_r.\beta_e}}} \tag{17}$$

If we compare (10) and (16) on one hand, (11) and (17) on the other hand, we can see that:

$$\beta = \frac{\beta_r - \beta_e}{1 - \beta_r.\beta_e} \tag{18}$$

Moreover, if the emitter and the receiver move each other according to velocities in opposite directions ($\alpha_e = 0$ and $\alpha_r = \pi$) or ($\alpha_e = \pi$ and $\alpha_r = 0$), then we will find:

$$\beta = \frac{\beta_r + \beta_e}{1 + \beta_r.\beta_e} \tag{19}$$

This concordance between the results provided by formula (8) when applied to the longitudinal case and the ones provided by the speed composition law of the classic theory [3], tends to give credit to our new proposed formula.

5 Introducing the Cosmic Microwave Background

The accepted relativistic Doppler effect formula convokes the only relative speed between an emitter and a receiver. As we have now defined a formula where the speed of the emitter and receiver are separated, we have to refer them to a privileged reference frame. Since the Cosmic Microwave Background [4] was discovered, it is possible to resort to such a reference and make such a separation.

For instance, the COBE Mission [5] was able to estimate the Heliocentric system velocity relative to the Cosmic Microwave Background and to provide the following values in terms of speed value and direction in a standard galactic coordinates system:

$$V_{Sun-CMB} = 369.5 km.s^{-1}$$
$$(l_{Sun-CMB}; b_{Sun-CMB}) = (264.4°; 48.4°)$$

6 Detailed analysis nearby the solar system

As an illustration of the use of the relativistic Doppler formula, we will focus on the Kapteyn's star, for which the speed parameters and redshift factor are well known and documented. The SIMBAD catalog [6] provides the following values for this star:

Table 1: Astrometric datas for Kapteyn's star

Name	D (kpc)	(l;b) (*degrees*)	z	V_{rad} ($km.s^{-1}$)	μ_α ($mas.y^{-1}$)	μ_δ ($mas.y^{-1}$)
Kapteyn's	$3.93.10^{-3}$	$(250°; -36°)$	$8.183.10^{-4}$	245	+6491	−5708

* Radial value of the speed $V_{rad} = c.z$ ($c \approx 3.10^5 km.s^{-1}$) :

$$V_{rad} = 245 km.s^{-1}$$

* According to the relation [7] $V_\alpha = 4.74.\mu_\alpha.D$:

$$V_\alpha = 121 km.s^{-1}$$

* According to the relation [7] $V_\delta = 4.74.\mu_\delta.D$:

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$$V_\delta = -106 km.s^{-1}$$

* Total value of the relative speed $V_{rel} = \sqrt{V_{rad}^2 + V_\alpha^2 + V_\delta^2}$:

$$V_{rel} = 293 km.s^{-1}$$

6.1 Geometric construction of the velocities relative to the CMB

According to the vector relation:

$$\overrightarrow{V}_{Star-CMB} = \overrightarrow{V}_{Star-Sun} + \overrightarrow{V}_{Sun-CMB}$$

With:

$$\overrightarrow{V}_{Star-Sun} = \overrightarrow{V}_{rel}$$
$$\overrightarrow{V}_{Star-CMB} = \overrightarrow{V}_{e}$$

We can deduce the following values for the Kapteyn's star's velocity relative to the Cosmic Microwave Background:

$$V_{Star-CMB} = 417 km.s^{-1}$$
$$(l_{Star-CMB}; b_{Star-CMB}) = (276°; 6.44°)$$

The angle (in degrees) between the Kapteyn's star's velocity relative to the CMB and the "Kapteyn-Sun" axis is given by:

$$\begin{aligned}\alpha_e = 180 - Arccos(&\cos(b).\cos(l).\cos(b_{Star-CMB}).\cos(l_{Star-CMB})\\ &+\cos(b).\sin(l).\cos(b_{Star-CMB}).\sin(l_{Star-CMB})\\ &+\sin(b).\sin(l_{Star-CMB}))\end{aligned}$$

That is:

$$\alpha_e \approx 131.1° \quad (20)$$

The angle (in degrees) between the Heliocentric system's velocity relative to the CMB and the "Kapteyn-Sun" axis is given by:

$$\begin{aligned}\alpha_r = 180 - Arccos(&\cos(b).\cos(l).\cos(b_{Sun-CMB}).\cos(l_{Sun-CMB})\\ &+\cos(b).\sin(l).\cos(b_{Sun-CMB}).\sin(l_{Sun-CMB})\\ &+\sin(b).\sin(l_{Sun-CMB}))\end{aligned}$$

That is:

$$\alpha_r \approx 94.6° \quad (21)$$

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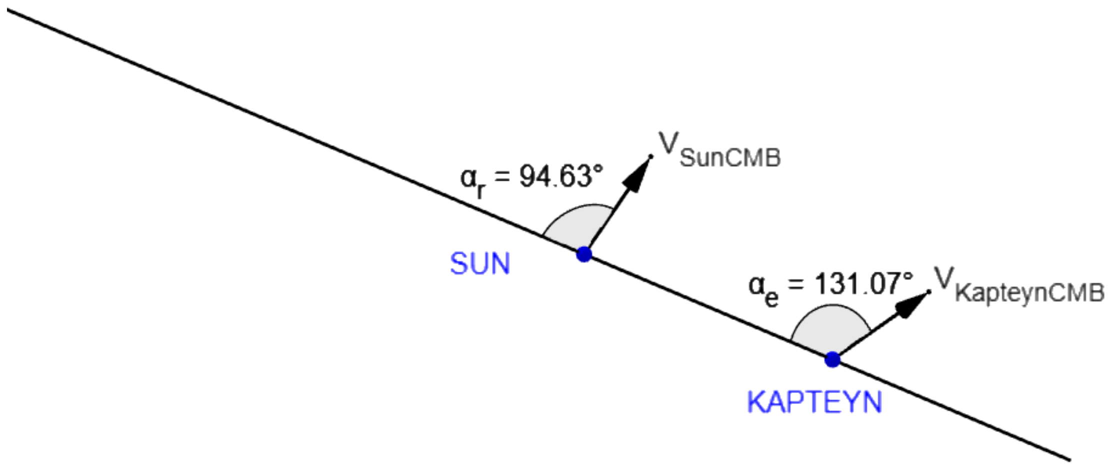

Figure 4: Geometric construction for the velocities of Kapteyn's star and the Heliocentric system relative to the CMB

6.2 Dichotomic calculus of the velocity parameters relative to the CMB

The relation between the Doppler ratio and the redshift factor is given by:

$$\frac{\lambda_r}{\lambda_e} = z + 1 \tag{22}$$

From formula (8), it is possible to establish a second order polynom where $\sin\alpha_e$ is the variable:

$$\beta_e^2.sin^2\alpha_e - \beta_e.\sqrt{1-\beta_e^2.sin^2\alpha_e}.\sqrt{1-sin^2\alpha_e}$$
$$+\frac{1}{2}.(z+1)^2.(1-\beta_e^2).(\sqrt{1-\beta_r^2.sin^2\alpha_r} - \beta_r.cos\alpha_r)^2 - \frac{1}{2}.(1+\beta_e^2) = 0 \tag{23}$$

Since we know from measurement the values of z, β_e ($V_{Star-CMB}/c$), β_r ($V_{Sun-CMB}/c$) and α_r, we get the following formula:

$$1.938.10^{-6}.sin^2\alpha_e - 1.392.10^{-3}.\sqrt{1-1.938.10^{-6}.sin^2\alpha_e}.\sqrt{1-sin^2\alpha_e} + 9.175.10^{-4} = 0$$

Let us consider the former polynom as a function of $\sin\alpha_e$ and call it: F. We can estimate $\sin\alpha_e$ graphically or, to be more precise, by dichotomic numeric calculus, such as:

$$F(sin\alpha_e) = 0$$

We get then for $\sin\alpha_e$:

$$\sin\alpha_e \approx 0.75$$

That is, for α_e (in degrees):

$$\alpha_e \approx 131.4° \tag{24}$$

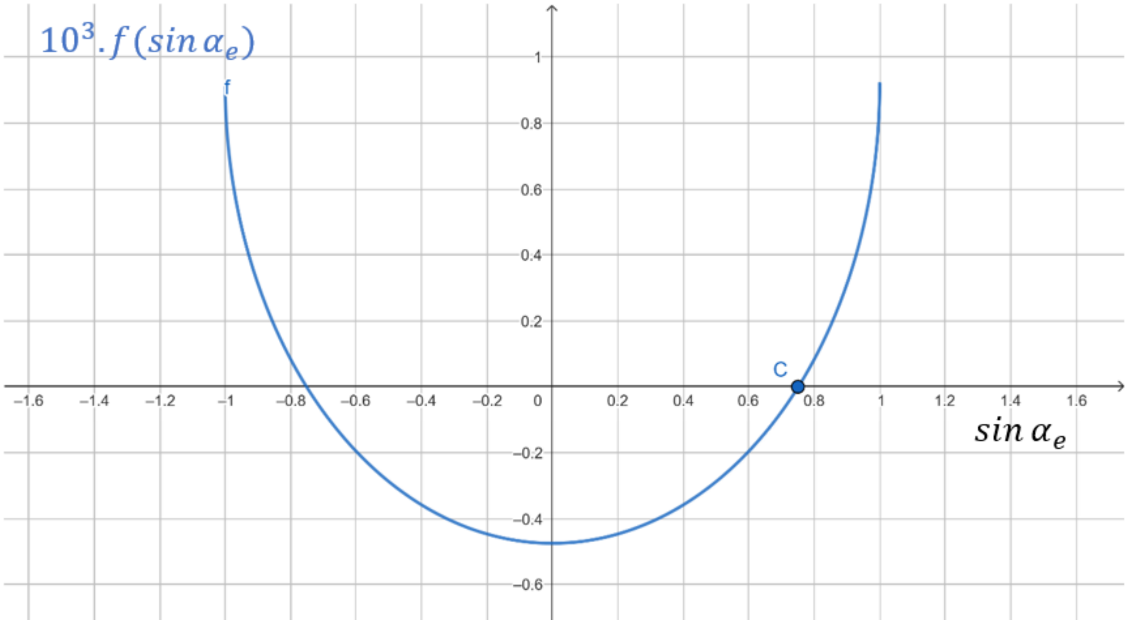

Figure 5:
Graphic representation of $F(\sin\alpha_e)$, amplified by a factor of 1000

There is a concordance between the value of α_e estimated by geometry and the one provided by dichotomic calculus based on the polynom (23). This concordance tends to support the quality of the latter, therefore the quality of the new Doppler formula (8).

By reproducing this operating method for a panel of stars whose proper motions are well documented, we can find again a concordance between the values of α_e geometrically estimated and the ones calculated according to the new Doppler formula.

Table 2: Estimation of α_e by geometric construction and dichotomic calculus

Name	z	(l;b)	V_e	α_e (geo)	α_e (dicho)
Kapteyn's	$8.183.10^{-4}$	(250;-36)	417.6	131.1	131.4
Barnard	$-3.672.10^{-4}$	(31;14)	478.5	112.7	113
Beta Hydri	$7.700.10^{-5}$	(304;-39)	352	89.36	88.5
Groombridge 1830	$-3.267.10^{-4}$	(168.5;73.8)	472.5	70.2	70.09
Lalande 21185	$-2.823.10^{-4}$	(185;65.4)	297.8	128.8	128

7 Extragalactic considerations

An innovative and recent study [8] suggests the existence of a global rotating movement within the Universe. Therefore, the recession velocities of extragalactic objects may not be strictly radial but would also include a tangential component. The new relativistic Doppler formula makes apparent some radial and tangential component to β_e indeed:

$$\beta_{//} = \beta_e.|\cos\alpha_e|$$

$$\beta_{\perp} = \beta_e.|\sin\alpha_e|$$

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If we assume a global rotation of the Universe and without prejudging the value of the Hubble constant H_0, it is possible to consider the latter as the combination of an axial component $H_{//}$ and an orthogonal one $H_\perp$:

$$H_0 = \sqrt{H_{//}^2 + H_\perp^2}$$

With:

$$H_{//} = H_0.|\cos \alpha_e|$$
$$H_\perp = H_0.|\sin \alpha_e|$$

And:

$$H_0 = \frac{V_e}{D}$$

If we convoke the Hubble tension, currently estimated at around 7% [8], and consider it as the difference between the total component H_0 and the radial component $H_{//}$ of the Hubble constant, then we obtain:

$$H_{//} = 0.93.H_0$$
$$H_\perp = 0.37.H_0$$

Which implies for α_e the following value in degrees:

$$\alpha_e = Arccos(-0.93)$$
$$\alpha_e = 180° \pm 21.6°$$

If we finally identify $H_{//}$ with the value determined from the ΛCDM model [9], and H_0 with the value estimated from the Doppler and Standard Candles method [10], then we also have:

$$H_0 \approx 73 km.s^{-1}.Mpc^{-1}$$
$$H_{//} \approx 67.9 km.s^{-1}.Mpc^{-1}$$
$$H_\perp \approx 26.8 km.s^{-1}.Mpc^{-1}$$

8 Conclusion

The relativistic Doppler effect formula is classicaly established by the application of the Lorentz transformations to the four-momentum of a photon. A top bottom consideration leads then from the relativistic case to the classic case when we have for the relative speed: $\beta << 1$

In this study, we rather make an assumption on the frequency of an electromagnetic emitter and receiver and the influence of movement on its properties according to the Lorentz factor g:

$$f_\beta = g.f_0$$

Where:

$$g = \sqrt{1 - \beta^2}$$

And f_0 is the fundamental or resting frequency of the emitter or receiver.

According to a bottom up approach, the relativistic Doppler effect is then establihed on the basis of the classic Doppler effect combined to our assumption on the frequency properties:

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$$\frac{\lambda_r}{\lambda_e} = \frac{\sqrt{1-\beta_r^2}}{\sqrt{1-\beta_e^2}} . \frac{\sqrt{1-\beta_e^2 . \sin^2\alpha_e} - \beta_e . \cos\alpha_e}{\sqrt{1-\beta_r^2 . \sin^2\alpha_r} - \beta_r . \cos\alpha_r}$$

Since this new formula makes possible to separate the speed of the emitter and the receiver by refering it to the Cosmic Microwave Background, it is possible to use it in modern astrophysics.

9 Copyright Notice

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