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Why is the Speed of Light Finite? The Theoretical Grounds for the Fact that the Speed of Light is Finite

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Abstract

The speed of light is finite, or the speed of any moving body including a photon cannot exceed a finite constant value. The reason is that the inertial frame in which we examine the motions of objects is not purely inertial. We assume that the spacetime around us in which the solar system lies is an inertial system. However, in the far distance there exist innumerable material bodies which exert the gravitational forces on the motions of the objects we examine. It is shown that, if it were not for those external bodies, the motion of a body would attain the infinite speed.

1 Introduction

The inertial spacetime is a space and time system that is influenced by no external actions such as gravitational forces. The only players in this spacetime should be the assembly of the bodies whose motions are to be examined. When we examine the motion of the solar system, for example, we assume such a frame of spacetime. The Newtonian mechanics is considered to hold in such a framework. We guess that Newton himself supposed such a system as his inertial system.

In the Newtonian mechanics, however, we do not know why there exists a limit of speed for every moving body or why the addition of two velocities is not the mathematical sum of the two velocity vectors. We have no theoretical reason for the finite limit of the velocity; only we have the observational fact for that.

As a result of the experiment by Michelson (1887), it was confirmed that the speed of light measured on whatever moving platform cannot exceed a certain constant value which is traditionally denoted by the letter c. This result is expressed symbolically by the following equation:

$$c + v = c \quad \text{(for any value of } v\text{)}. \tag{1}$$

After the discovery of this fact and the many subsequent experiments for verification, the Newtonian mechanics has been drastically modified and a new system of mechanics on the relativistic principle has replaced it, achieving a brilliant development since then.

Meanwhile, it seems that the theoretical reason why the limit of speed exists has never been proposed nor even the question why the speed of light is finite has ever been asked as a decent scientific problem.

Surely there is a close relation between the finiteness of the light speed and the relativity theory. However, the truth in this regard is that the relativity theory was produced by the observational fact that the speed of light is finite but not that the relativity theory derived the finiteness of the light speed theoretically.

In the present study it is attempted to ask this question of why the speed of light is finite seriously and to give a tentative answer to it.

The speed of light has a value of c in the actual spacetime around us where the solar system is placed. In the present study we investigate how the behaviour of bodies would change if there were no objects in the universe other than the members to be investigated. The former universe is considered to be a quasi-inertial system while the latter a pure inertial system.

We pay attention to a void space in the actual universe which is filled with material bodies. In some of such void spaces the attractions by the gravitation from all directions may balance and no force works there. Such a void space could not be distinguished from the pure inertial spacetime. In general void spaces, however, the gravitational attractions from all directions do not balance and some uniform and constant attraction or acceleration is considered to remain there.

In such a void, if the remaining uniform acceleration extends to sufficiently large spans of space and time, the Newtonian mechanics would hold as well. That is, such a space and time will form a quasi-inertial spacetime. The actual spacetime we recognize around us is inertial in this sense and we call it our inertial spacetime.

In the present study, we examine the nature of a void space where a constant acceleration prevails throughout it and investigate the behaviour of the objects in it to find out how the velocities of objects are different from those in the pure inertial frame.

But, before that, we show in the next section that the speed of a moving body can attain a speed larger than c by a simple calculation.

2 The velocity can be larger than c

When we investigate the motions for a group of mass bodies in an inertial spacetime, we usually suppose that the framework in which the members of the group are moving contains no other material bodies to exert the action on them other than the members themselves. For example, when we investigate the motions of the solar system, we implicitly suppose such an inertial system as the framework.

Contrary to this, however, in the far distance there exist innumerable material bodies which exert the gravitational forces on the motions of the objects we examine. Meanwhile, in our actual spacetime there exists the observational fact that the speed of a moving body including a photon cannot exceed the speed with a constant quantity denoted by c.

In this section we show that the speed of a body can exceed c if it were not for the external bodies in the framework, after a simple consideration.

Kubo (2019,2022) showed that, as a result of the study on the motion of the planet with respect to the sun and that of the sun with respect to the planet, the scales of space and time are changed around a mass body. The formula of the change is as follows:

As for the time scale, let the nominal values for the same time duration at the same place be denoted by τ when it is not affected by the gravitation of a body and by τ' when it is affected. Then, the relation between τ' and τ is given by

$$\tau' = (1 - \frac{2\mu}{c^2 r})\tau, \tag{2}$$

where $\mu = GM$, with G being the universal gravitational constant and M the mass of the body.

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Also the quantity c is the speed of light in our inertial system and r is the distance from the mass body to the position of the observer. Eq. (2) shows that the time nominal value τ not affected by the gravitation of a mass is larger than τ' affected by the gravitation, that is, τ' elapses more slowly than τ.

Next, as for the spatial scale, the problem is more complicated because the scale depends on the direction. Consider a sphere with the body of the gravitational source at the centre, and let the observer be located on the spherical surface. Let r denote the distance from the central body to the observer along the radial direction and l the distance along the surface. Then we have the following equations Kubo (2019,2022), together with the equation for the time scale:

$$\begin{aligned} r' &= r - \frac{\mu}{c^2} = \left(1 - \frac{\mu}{c^2 r}\right) r, \\ dr' &= dr, \\ d\theta' &= d\theta, \\ dl' &= r' d\theta' = \left(1 - \frac{\mu}{c^2 r}\right) r d\theta = \left(1 - \frac{\mu}{c^2 r}\right) dl, \\ dt' &= \left(1 - \frac{2\mu}{c^2 r}\right) dt, \end{aligned} \tag{3}$$

where the values with the prime($'$) and that without it mean the nominal values of affected by the gravitation of the central body and not affected as for the same quantities, respectively.

In Eq. (3), the first and the fifth equations are the direct result from the discussion on the motions of the planet with respect to the sun and of the sun with respect to the planet. The second equation is obtained from the first one and the third one comes from the demand for isotropy. The fourth equation results from the first and the third ones.

Next let us consider the change in the length in a general direction. Let the nominal length in a general direction in the spacetime affected by the gravitational source and that not affected be denoted by σ' and σ, respectively. Then,

$$\sigma = \sqrt{dr^2 + dl^2} \quad \text{and} \quad \sigma' = \sqrt{dr'^2 + dl'^2}, \tag{4}$$

the value of σ/σ' depending on the angle between the the direction in which the distance σ is measured and the direction from the gravitation source to the observer. For a distance σ along the radius vector,

$$dr' = dr, \tag{5}$$

and for a distance along the surface of the sphere,

$$dl' = (1 - \frac{\mu}{c^2 r}) dl < dl \tag{6}$$

Thus, for σ in a general direction we have

$$\sigma' \leq \sigma. \tag{7}$$

Then, as for the nominal values for the same velocity, v' in the spacetime affected by the gravitation and v not affected, we have the following equation:

$$v' = \frac{\sigma'}{\tau'} = \frac{\sigma'}{(1 - 2\mu/(c^2 r))\tau} \leq \frac{\sigma}{(1 - 2\mu/(c^2 r))\tau} = \frac{v}{1 - 2\mu/(c^2 r)}. \tag{8}$$

Eq. (8) means that, if there exists an outer body, the velocity v' affected by the body is smaller than the velocity v not affected, at the ratio equal to or larger than $1/\left(1 - 2\mu/(c^2 r)\right)$.

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Inversely, if an outer body were removed, the velocity v' would become larger than v at the same ratio, that is,

$$v = \frac{v'}{1 + 2\mu/(c^2 r)}. \tag{9}$$

If two outer bodies were removed, the velocity would come to v'', as

$$v'' \geq (1 - \frac{2GM_2}{c^2 r_2})v' = (1 + \frac{2GM_2}{c^2 r_2})(1 + \frac{2GM_1}{c^2 r_1})v, \tag{10}$$

thus v'' being further larger than v'.

Finally, we consider the case where all the outer bodies have been removed. We write the velocity in this case as $\acute{v}$, which is as a matter of fact the velocity in the pure inertial system. Then,

$$\acute{v} \geq \left(\prod_{i=1}^{N} \left(1 + \frac{2GM_i}{c^2 r_i} \right) \right) v, \tag{11}$$

with N being the total number of the outer bodies. The velocity $\acute{v}$ may be large endlessly. Even it might be infinite. If the velocity is that of a photon, the velocity of light in the pure inertial system $\acute{v}$ can be larger than c.

It is not easy, however, to obtain the explicit expression for the relation between v and $\acute{v}$, by continuing to calculate after Eq. (11). So, in the followings we will attempt a different approach for obtaining it.

3 A void space in the universe filled with materials

Consider a universe in which no material bodies exists. Such a universe would supply a completely inertial space. If, for example, a group of members are placed in this universe, the Newtonian mechanics would give a complete solution to the motions of the members. We can regard such a spacetime as completely inertial and may call it a pure inertial system. However, the actual spacetime around us is far from this. We are surrounded by innumerable material bodies in the far away distance. We live only in a void space in the universe full of materials each of which causes the attractive forces.

Even in such a universe there are many void spaces, and if in one of the voids a uniform acceleration prevails throughout it, the void could be regarded as practically inertial. In fact, if the accelerations from all the external bodies completely balances and no acceleration referred to the pure inertial system acts there, the spacetime would give quite the same inertial system as the pure inertial one.

Even not so, if the acceleration that remains in the void is practically uniform, say $\vec{a}$, covering sufficiently large spans of space and time, the spacetime could be regarded as practically inertial and the Newtonian mechanics would practically hold there.

Thus, a void space in the universe which is full of materials can be almost inertial. We actually live in one of such void spaces. We call it our void space and the spacetime that sticks on it our actual inertial system.

4 The conversion formula between two coordinate systems mutually accelerating

Concerning the conversion of coordinates between two coordinate systems that are accelerating to each other we can investigate it by the transformation formula explained in Zhukov (1961), Kubo (2019) or Kubo (2022).

Note that the formula is essentially for one dimensional space and only treats the motion or its component along the same direction as that of the acceleration. However, taking it into account that the motion perpendicular to the direction of the acceleration is not affected by the acceleration, it would be easy to extend the problem to the one in general directions or in three dimensions.

The formula appearing in Zhukov (1961), etc. says as follows:

Of the two mutually accelerating coordinate systems, let A-system be expressed by (x, y) and B-system by (x', t'). Of the two systems we consider A-system to be the reference coordinate system.

The reference coordinate system is not necessarily a pure inertial system. Rather, the formula assumes that our actual inertial system is the reference system.

It is supposed that the two systems are quite the same and were completely coincident at the initial time $t = 0$ and $t' = 0$, and their relative velocity $u = 0$ at that instant. The origin of B-system is supposed to move with respect to its instantaneously comoving coordinate system with a constant acceleration α. On these conditions the conversion formula between (x, t) and (x', t') is as follows:

$$\begin{aligned} x' &= \frac{c^2}{\alpha} \log \frac{\alpha}{c^2} \sqrt{\left(x + \frac{c^2}{\alpha}\right)^2 - c^2 t^2}, \\ t' &= \frac{c}{2\alpha} \log \frac{x + ct + \dfrac{c^2}{\alpha}}{x - ct + \dfrac{c^2}{\alpha}}, \end{aligned} \tag{12}$$

and the inverse transformation is:

$$\begin{aligned} x &= \frac{c^2}{\alpha} e^{\frac{\alpha}{c^2} x'} \cosh \frac{\alpha}{c} t' - \frac{c^2}{\alpha}, \\ t &= \frac{c}{\alpha} e^{\frac{\alpha}{c^2} x'} \sinh \frac{\alpha}{c} t'. \end{aligned} \tag{13}$$

The derivation of Eqs. (12) and (13) is given in the appendix 9. By the way, the formula above is fully used in obtaining Eq. (3).

5 The explicit expression for the relation between v and $\acute{v}$

In the discussion below, we set A- and B- systems as follows (FIG.1 illustrates the relation between both systems):

The spatial origins of both systems are supposed to have been coincident with each other at the time $t = 0$ and $t' = 0$. The spatial origin of B-system moves with respect to its instantaneously comoving coordinate system, or with respect to itself, with the acceleration α. Also, all the points fixed in B-system moves with respect to its instantaneously comoving coordinate system with the acceleration α as well, thus the relative positions of all the points in B-system being unchanged. But the acceleration of any point relative to A-system depends on its coordinates (x', t').

Designating A-system as the reference system, let its acceleration and velocity have been respectively the same as those of our void space at $t = t_0 = 0$ in the time of A-system.

Meanwhile let the spatial origin of B-system O′ have had the acceleration of $-\vec{a}$ with respect to itself, with $\vec{a}$ being the remaining acceleration in our void space, whereas its velocity have been the same as that of our void space at $t' = t'_0 = 0$ in the time of B-system, consequently both systems having been instantaneously at rest to each other at $t = 0$ and $t' = 0$.

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This setting corresponds to applying the formula Eqs. (12) and (13) substituting $-a$ for α, and it results in that the acceleration of B-system with respect to the pure inertial system is 0, that is, B-system is a pure inertial spacetime.

Also, we notice this setting implies that A-system is identical to B-system at $t' = t'_0$ except for the acceleration.

Now, we consider a point P moving in A-system and examine the velocity of the point in B-system. Let point P in A-system be called point Q in B-system, and let the velocity of P in A-system be v while the velocity of Q referred to the spatial origin of B-system O$'$ be v'.

Choose B-system so that point Q is at rest with respect to O$'$ at $t = t'_1$. We can always find such a B-system. In fact, if point Q in some spacetime moves along a line $x' =$ const. with respect to O$'$ at $t = t'_1$ in it, then the spacetime is qualified for a B-system (FIG.1). There are innumerable such points. If a point is qualified, all the points on the line with the same value of t' are also qualified. Suppose that t' satisfies Eq. (16) below, i.e., $t' = (c/\alpha)\tanh^{-1}(v/c)$ at $t' = t'_1$.

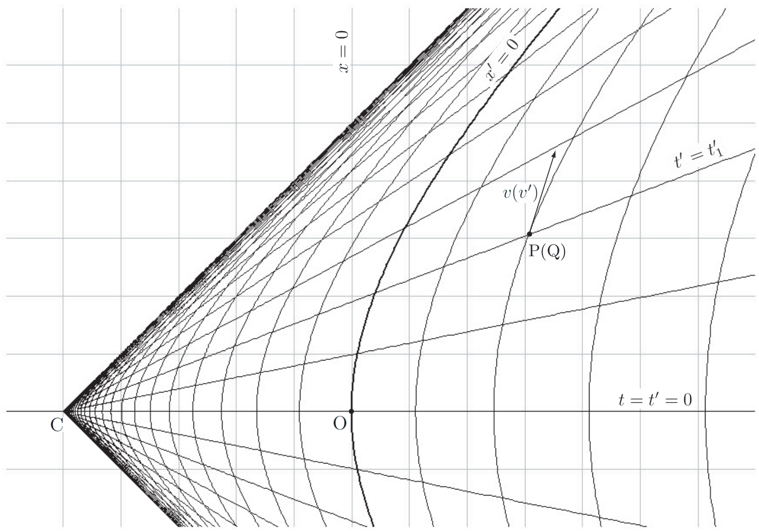

Figure 1: Coordinates frame consisting of the spatial and time lines of B-system (black lines) drawn in the diagonal coordinates grid of A-system (gray lines). x-axis is taken in the direction of the acceleration α. The bold line curve is the locus of the spatial origin of B-system. The velocity of the point P(Q) referred to A-system at $t = t_0$ is $v = dx/dt$ and that referred to B-system at $t' = t'_0$ is $v' = 0$.

The velocity v in terms of the coordinates of A-system is

$$v = \frac{dx}{dt}. \tag{14}$$

Next let us express the right-hand side quantity in Eq. (14) in terms of the coordinates of B-system. Since the point is at rest referred to the spatial origin of B-system at the time $t' = t'_1$, we have $x' =$ const. or $dx' = 0$. Then, from Eq. (13),

$$\begin{aligned} dx &= c e^{\frac{\alpha}{c^2}x'} \sinh\frac{\alpha}{c}t' dt', \\ dt &= e^{\frac{\alpha}{c^2}x'} \cosh\frac{\alpha}{c}t' dt', \end{aligned} \tag{15}$$

and from this equation we have at once

$$v = \frac{dx}{dt} = c\tanh\frac{\alpha}{c}t', \tag{16}$$

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where $t' = t'_1 - t'_0$ is the time in B-system that has elapsed since the moment t'_0.

More exactly, v is the velocity of P in A-system at the time $t = t_0 = 0$. We have to compare it with the velocity of point Q in B-system at the same instant, i.e. at the time $t' = t'_0 = 0$. However, as we noticed above, it is equal to the velocity of point P in A-system at the time $t = t_0$, which is v itself. In addition, as it is expressed in terms of the coordinates of B-system, the right-hand side expression in Eq. (16) is just what we have intended to find.

Since O$'$ at t'_0 is at rest with respect to the pure inertial system, O$'$ at the time t'_1 is moving at some velocity, say v'_0, with respect to B-system at $t' = t'_0$.

v' is the velocity of point Q referred to O$'$ at $t' = t'_1$ and $v' = 0$. Then, the velocity of point Q with respect to O$'$ at $t' = t'_0$ is $v'_0 + v' = v'_0$.

However, since B-system is a pure inertial system, the Newtonian mechanics holds in this spacetime. Then, we have $v'_0 = \alpha(t'_1 - t'_0)$ there. Thus, v'_0 is the velocity of Q in a pure inertial system at the time $t' = t'_1$. And v'_0 is nothing else but $\acute{v}$ which we examined in Section 2.

Thus, we have

$$\frac{v}{c} = \tanh \frac{\acute{v}}{c}. \tag{17}$$

Eq. (17) is the relation between v and $\acute{v}$, i.e., the velocities of the same moving point measured in our practical inertial system and in a pure inertial system, respectively.

6 Conclusion

The inertial spacetime we recognize around us which contains the solar system whose member's motions we study theoretically is not a pure inertial system because it is affected by the gravitational forces by the innumerable outer bodies other than the objects we treat as the subject of our study. But this apparent inertial system, i.e., our actual inertial system, functions as a kind of inertial system and each member of bodies in it makes a similar behaviour to that in a pure inertial system.

However, the motion in our actual inertial system is not rigorously the same as that in the pure inertial system. In the present study, the velocity of a particle v observed in our practical inertial system is proved to be observed as $\acute{v}$ in the pure inertial system with the relation between both being given by Eq. (17).

For example, the speed of light c observed in our practical inertial system comes to ∞ in the pure inertial system. Inversely, the speed of the value ∞ in the pure inertial system comes to c in our system.

Thus, we see that Eq. (1) stems from the mathematical equation: $\infty + a = \infty$ (for any value of a).

Also we see another example. In the pure inertial system where we consider that the Newtonian mechanics holds theoretically, we have

$$\acute{v}_1 + \acute{v}_2 = \acute{v}_3 \tag{18}$$

where $\acute{v}_3$ is the mathematical sum of $\acute{v}_1$ and $\acute{v}_2$. Let us translate the formula (18) into the formula in our practical inertial system:

The sum of two velocities in our inertial system is the relativistic one. So, writing it as $v_1 \oplus v_2$, we have

$$\begin{aligned} v_1 \oplus v_2 = c \tanh \frac{\acute{v}_1 + \acute{v}_2}{c} &= c \left(\tanh \frac{\acute{v}_1}{c} + \tanh \frac{\acute{v}_2}{c} \right) \Big/ \left(1 + \tanh \frac{\acute{v}_1}{c} \cdot \tanh \frac{\acute{v}_2}{c} \right) \\ &= \frac{v_1 + v_2}{1 + (v_1/c)(v_2/c)}, \end{aligned} \tag{19}$$

a familiar formula in the relativity theory.

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Finally, the present author does not think that the present study is a very rigorous one and he expects more rigorous or more elegant solutions will be presented in the future.

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8 Copyright Notice

9 Appendix: Nature of mutually accelerating coordinate systems

In this appendix, we review the formula for the coordinates transformation between mutually accelerating two coordinate systems, following Zhukov (1961) but supplementing some facts not mentioned in it.

We consider a spacetime S-system to be the reference system and S′-system is accelerating referred to it. It seems to be implied that S-system is a pure inertial system, but it is not correct as we see below soon.

Taking x-axes of both systems in the direction of the acceleration, let S-system be represented by (x, t). Then, consider a point P moving with respect to its instantaneously comoving coordinate system with the acceleration α, or, in other words, with the acceleration α with respect to itself. Let the velocity of P relative to S-system be u at the instant considered.

S′-system is such a system that the positions of all the points in it relative to the point P are kept constant. Let it be represented by (x', t'). Suppose that S-system and S′-system coincided with each other completely at the initial epoch.

Let the velocity of P referred to S-system at some instant t_1 be u_1 and the velocity at the instant $t_2 = t_1 + \Delta t'$ be u_2. Then, we have

$$u_2 = \frac{u_1 + \alpha \Delta t'}{1 + u\alpha \Delta t'/c^2}, \tag{20}$$

where $\Delta t'$ is the time interval expressed in terms of the time in S′- system that corresponds to Δt in (x, t) system. Then, the acceleration of the point P with respect to S-system is obtained by

$$\frac{du}{dt} = \lim_{\Delta t \to 0} \frac{u_2 - u_1}{\Delta t} = \alpha \left(1 - \frac{u^2}{c^2}\right)^{3/2}, \tag{21}$$

where $\Delta t' = \sqrt{1 - u^2/c^2}\Delta t$ is made use of, which is obtained by putting $\Delta x' = 0$ in the Lorentz transformation formula from $(\Delta x', \Delta t')$ to $(\Delta x, \Delta t)$. As a matter of fact, Eq. (21) shows that S-system is our actual inertial system but not the pure inertial system.

Integrating Eq. (21) we have

$$u = \frac{\alpha t}{\sqrt{1 + \alpha^2 t^2/c^2}}, \tag{22}$$

where we suppose $u = 0$ at $t = 0$. As for the motion of the point P with respect to S-system, we have

$$x = \int_0^t u dt = \int_0^t \frac{\alpha t}{\sqrt{1 + \alpha^2 t^2/c^2}} dt = \frac{c^2}{\alpha} \left(\sqrt{1 + \frac{\alpha^2}{c^2} t^2} - 1 \right), \tag{23}$$

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similarly supposing $x = 0$ at $t = 0$. This equation shows that the locus of the point P is one branch of such a hyperbola as its centre is at $(x = -c^2/\alpha, t = 0)$, which we call the point C, and the equations of its asymptotes are $x = \pm ct - c^2/\alpha$. In case of $\alpha > 0$, it is the right-hand side branch of the centre. The locus for Eq. (23), which is the curve for $x' = 0$, is shown in a bold line curve in FIG. 2.

The time τ that elapses at the point P, which is the proper time of the point, is given by

$$\begin{aligned}\tau &= \int_0^t dt' = \int_0^t \sqrt{1 - \frac{u^2}{c^2}} dt = \int_0^t \frac{dt}{\sqrt{1 + \alpha^2 t^2/c^2}} \\ &= \frac{c}{\alpha} \log\left(\sqrt{1 + \frac{\alpha^2}{c^2}t^2} + \frac{\alpha}{c}t\right) = -\frac{c}{\alpha} \log\left(\sqrt{1 + \frac{\alpha^2}{c^2}t^2} - \frac{\alpha}{c}t\right).\end{aligned} \tag{24}$$

In deriving this equation, we make use of the supposition that the spatial and time scales in S- and S′- systems (in the vicinity of P as for S′-system) are the same. In this case we have $dt' = \sqrt{1 - u^2/c^2} dt$, which is obtained by putting $dx' = 0$ in the Lorentz transformation from (dx', dt') to (dx, dt) since the integration is carried out along the curve for $x' = 0$.

Now, let us take a point $X(\bar{x}, \bar{t})$ in the right-hand side area of the curve for $x' = 0$ and draw two lines with the inclination $\mp c$ from this point, respectively expressed by the equations $x - \bar{x} = -c(t - \bar{t})$ and $x - \bar{x} = c(t - \bar{t})$, and let the coordinates of the points where the lines intersect the curve for $x' = 0$ be (x_1, t_1) and (x_{-1}, t_{-1}), respectively. Then, from these equations and Eq. (23) we have

$$\begin{aligned}\bar{x} - c(t_1 - \bar{t}) &= \frac{c^2}{\alpha}\left(\sqrt{1 + \frac{\alpha^2}{c^2}t_1^2} - 1\right), \\ \bar{x} + c(t_{-1} - \bar{t}) &= \frac{c^2}{\alpha}\left(\sqrt{1 + \frac{\alpha^2}{c^2}t_{-1}^2} - 1\right).\end{aligned} \tag{25}$$

Add $(c^2/\alpha + ct_1)$ or $(c^2/\alpha - ct_{-1})$ to both sides. And then, writing τ corresponding to t_1 and t_{-1} as τ_1 and τ_{-1}, respectively, we have from Eqs. (24) and (25),

$$\begin{aligned}\tau_1 &= \frac{c}{\alpha} \log\left(\frac{\alpha}{c^2}\bar{x} + \frac{\alpha}{c}\bar{t} + 1\right), \\ \tau_{-1} &= -\frac{c}{\alpha} \log\left(\frac{\alpha}{c^2}\bar{x} - \frac{\alpha}{c}\bar{t} + 1\right),\end{aligned} \tag{26}$$

If we write as $(\tau_1 + \tau_{-1})/2 = t'$, the point X is located at a point where the time in S′-system is t'. All X's with the same value of t' form a line in S-system, which is expressed by the following equation:

$$t' = \frac{c}{2\alpha} \log \frac{\frac{\alpha}{c^2}x + \frac{\alpha}{c}t + 1}{\frac{\alpha}{c^2}x - \frac{\alpha}{c}t + 1}. \tag{27}$$

Also, $c(\tau_1 - \tau_{-1})/2$ means the distance x' of the point X from the point on the curve for $x' = 0$ corresponding to the time t'. All X's with the same value of x' form a line in S-system that is expressed by the equation:

$$x' = \frac{c^2}{\alpha} \log \sqrt{\left(\frac{\alpha}{c^2}x + 1\right)^2 - \frac{\alpha^2}{c^2}t^2}, \tag{28}$$

with x' being 0 on the curve $x' = 0$.

Concerning the points in the area on the left-hand side of the curve for $x' = 0$, a similar argument holds good, as well. Equations (27) and (28) are Eq. (12).

Eq. (27) stands for a straight line that passes the point C for every value of t', and different values of t' form a set of straight lines. They construct spatial lines (each line representing all the spatial points at a common time) of S′-system.

Eq. (28) stands for a hyperbola for every value of x' whose centre and asymptotes are common to those of the curve for $x' = 0$, only the right-hand side branch of each hyperbola being significant. Different values of x' form a set of hyperbolas which are similar to each other with the centre of similarity at the point C. They construct time lines (each line representing all the time points for some spatial coordinate) of S′-system. Thus the groups of the spatial lines and of the time lines form the coordinates frame of S′-system drawn in S-system as shown in FIG.2. This frame ought to be recognized as an orthogonal frame by an observer in S′-system (cf. FIG.3).

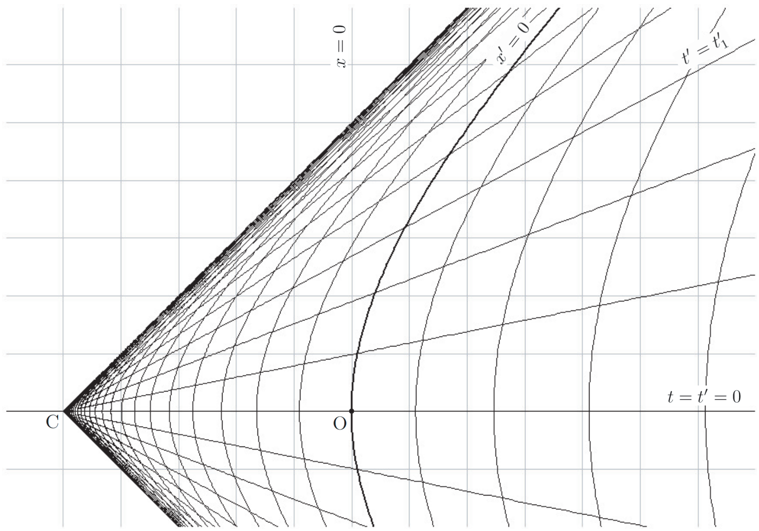

Figure 2: Coordinates frame consisting of the spatial and time lines of S′-system (black lines) drawn in the grid of S-system (gray lines). The bold line curve is the world line of the spatial origin of S′-system that moves with a constant acceleration α with respect to itself, drawn in S-system.

From Eq. (28) we have

$$dx' = \frac{\left(\frac{\alpha}{c^2}x + 1\right)dx - \alpha t dt}{\left(\frac{\alpha}{c^2}x + 1\right)^2 - \frac{\alpha^2}{c^2}t^2}. \tag{29}$$

In this equation, if $dx' = 0$, the quantity dx/dt means the velocity u of a point moving along the line with $x' = \text{const}$. Thus we have

$$u = \frac{\alpha t}{(\alpha/c^2)x + 1}. \tag{30}$$

Then,

$$dx' = \frac{\frac{\alpha}{c^2}x + 1}{\left(\frac{\alpha}{c^2}x + 1\right)^2 - \frac{\alpha^2}{c^2}t^2}\left(dx - u dt\right). \tag{31}$$

On the other hand, from Eq. (30) we have

$$1-\frac{u^2}{c^2}=\frac{\left(\frac{\alpha}{c^2}x+1\right)^2-\frac{\alpha^2}{c^2}t^2}{\left(\frac{\alpha}{c^2}x+1\right)^2}. \tag{32}$$

Hence, using Eq. (28),

$$dx'=e^{-\frac{\alpha}{c^2}x'}\frac{dx-udt}{\sqrt{1-u^2/c^2}}. \tag{33}$$

And after a similar calculation,

$$dt'=e^{-\frac{\alpha}{c^2}x'}\frac{dt-(u/c^2)dx}{\sqrt{1-u^2/c^2}}. \tag{34}$$

Eqs. (33) and (34) show that the Lorentz transformation do not hold for the coordinate transformation between mutually accelerating systems but the transformation must be conducted following these equations. Using Eqs. (28) and (30) again, Eqs. (33) and (34) are also written as

$$dx'=\frac{dx-udt}{[(\alpha/c^2)x+1](1-u^2/c^2)} \quad \text{and} \quad dt'=\frac{dt-(u/c^2)dx}{[(\alpha/c^2)x+1](1-u^2/c^2)}. \tag{35}$$

By solving Eq. (12) as for t and x, we obtain the inverse transformation formula Eq. (13). In this case, we notice that Eq. (13) is not attained by exchanging the variables (x,t) and (x',t') and replacing α by $-\alpha$ in Eq. (12), since Eq. (12) and Eq. (13) are not symmetrical. Meanwhile, we can show that Eq. (12) is derived directly as well, in the same way as Eq. (12) was derived in the above.

The point P, which is the same point P as we examined above, is moving in S-system. Let its instantaneous velocity be u_1 at some moment t'_1 in the time of S$'$-system. Meanwhile, introduce an intermediate system moving with the constant velocity u_1 with respect to S-system. This system does not have an acceleration to S-system and the point P is at rest instantaneously in this system.

On the other hand, consider a point P$'$ fixed in S-system. The velocity u' of P$'$ with respect to the intermediate system at t'_1 is equal to $-u_1$. Let it be u'_1, that is $u'_1=-u_1$. The instantaneous velocity of P$'$ relative to P or to S$'$-system is the same.

The point P is accelerating with respect to itself with the acceleration α. At the time $t'_2=t'_1+\Delta t'$, the velocity of the intermediate system with respect to the point P is $-\alpha\Delta t'$ and consequently the velocity of the point P$'$ referred to the point P or to S$'$-system is given by

$$u'_2=\frac{-u'_1-\alpha\Delta t'}{1+\alpha\Delta t' u'_1/c^2}. \tag{36}$$

Then, the acceleration of P$'$ with respect to S$'$-system is obtained as

$$\frac{du'}{dt'}=\alpha'=\lim_{\Delta t'\to 0}\frac{u'_2-u'_1}{\Delta t'}=\lim_{\Delta t'\to 0}\frac{-\alpha\Delta t'(1-u'^2/c^2)}{\Delta t'(1+u'\alpha\Delta t'/c^2)}=-\alpha\left(1-\frac{u'^2}{c^2}\right), \tag{37}$$

where u' is dx'/dt'. Note that Eq. (37) contrasts with Eq. (21).

From Eq. (37) we have

$$u'=\int_0^{t'}\alpha' dt'=-c\tanh\frac{\alpha}{c}t', \tag{38}$$

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where we suppose $u' = 0$ at $t' = 0$. Then, we have the x-coordinate of the point P$'$ with respect to S$'$-system as following:

$$x' = \int_0^{t'} u'dt' = -\frac{c^2}{\alpha} \log \cosh \frac{\alpha}{c} t', \tag{39}$$

if we suppose $x' = 0$ at $t' = 0$. The locus given by Eq. (39) is the curve for $x = 0$ in FIG.3.

Next, let us consider the time that elapses at the point P$'$, which is not other than the time t in the S- system, and write it as τ'. In this case, it is the time along the line for $x =$ const. and therefore by putting $dx = 0$ in Eq. (34), we have

$$dt = e^{\frac{\alpha}{c^2}x'} \sqrt{1 - \frac{u^2}{c^2}} dt'. \tag{40}$$

Then, using Eqs. (38), (39) and (40), τ' is expressed by t' as

$$\tau' = \int_0^{t'} dt = \int_0^{t'} \mathrm{sech}^2 \frac{\alpha}{c} t' dt' = \frac{\alpha}{c} \tanh \frac{\alpha}{c} t', \tag{41}$$

and this is written as

$$\begin{aligned} \tau' &= \frac{\alpha}{c} \frac{e^{\frac{\alpha}{c}t'} - e^{-\frac{\alpha}{c}t'}}{e^{\frac{\alpha}{c}t'} + e^{-\frac{\alpha}{c}t'}} \\ &= \frac{\alpha}{c} \frac{(e^{\frac{\alpha}{c}t'} + e^{-\frac{\alpha}{c}t'}) - 2e^{-\frac{\alpha}{c}t'}}{e^{\frac{\alpha}{c}t'} + e^{-\frac{\alpha}{c}t'}} = \frac{\alpha}{c} \frac{2e^{\frac{\alpha}{c}t'} - (e^{\frac{\alpha}{c}t'} + e^{-\frac{\alpha}{c}t'})}{e^{\frac{\alpha}{c}t'} + e^{-\frac{\alpha}{c}t'}} \\ &= \frac{\alpha}{c} \left[1 - \left(e^{\frac{\alpha}{c}t'} \cosh \frac{\alpha}{c} t' \right)^{-1} \right] = \frac{\alpha}{c} \left[\left(e^{-\frac{\alpha}{c}t'} \cosh \frac{\alpha}{c} t' \right)^{-1} - 1 \right]. \end{aligned} \tag{42}$$

In a similar way to what was used in deriving Eqs. (27) and (28), we take a point $(\bar{x}', \bar{t}')$ in the right-hand side area of the curve for $x = 0$, and draw two lines with the inclination $\mp c$ from this point, respectively expressed by the equations $x' - \bar{x}' = -c(t' - \bar{t}')$ and $x' - \bar{x}' = c(t' - \bar{t}')$, and let the coordinates the points where the lines intersect with the curve for $x = 0$ be (x'_1, t'_1) and (x'_{-1}, t'_{-1}), respectively.

Then, from Eq. (39) and these equations we have

$$\begin{aligned} \bar{x}' - c(t'_1 - \bar{t}') &= -\frac{c^2}{\alpha} \log \cosh \frac{\alpha}{c} t'_1, \\ \bar{x}' + c(t'_{-1} - \bar{t}') &= -\frac{c^2}{\alpha} \log \cosh \frac{\alpha}{c} t'_{-1}. \end{aligned} \tag{43}$$

Add to both sides $[ct'_1 - (c^2/\alpha) \log(\exp(-\alpha t'_1/c))]$ or $[-ct'_1 - (c^2/\alpha) \log(\exp(-\alpha t'_{-1}/c))]$, respectively. Then,

$$\begin{aligned} \frac{\alpha}{c^2} \bar{x}' + \frac{\alpha}{c} \bar{t}' &= -\log \left(e^{-\frac{\alpha}{c}t'_1} \cosh \frac{\alpha}{c} t'_1 \right) \\ \frac{\alpha}{c^2} \bar{x}' - \frac{\alpha}{c} \bar{t}' &= -\log \left(e^{\frac{\alpha}{c}t'_{-1}} \cosh \frac{\alpha}{c} t'_{-1} \right). \end{aligned} \tag{44}$$

Writing τ' corresponding to t'_1 and t'_{-1} as τ'_1 and τ'_{-1}, respectively, we have from Eqs. (42) and (44)

$$\begin{aligned} \tau'_1 &= \frac{c}{\alpha} \left(e^{\frac{\alpha}{c^2}\bar{x}' + \frac{\alpha}{c}\bar{t}'} - 1 \right), \\ \tau'_{-1} &= \frac{c}{\alpha} \left(1 - e^{\frac{\alpha}{c^2}\bar{x}' - \frac{\alpha}{c}\bar{t}'} \right). \end{aligned} \tag{45}$$

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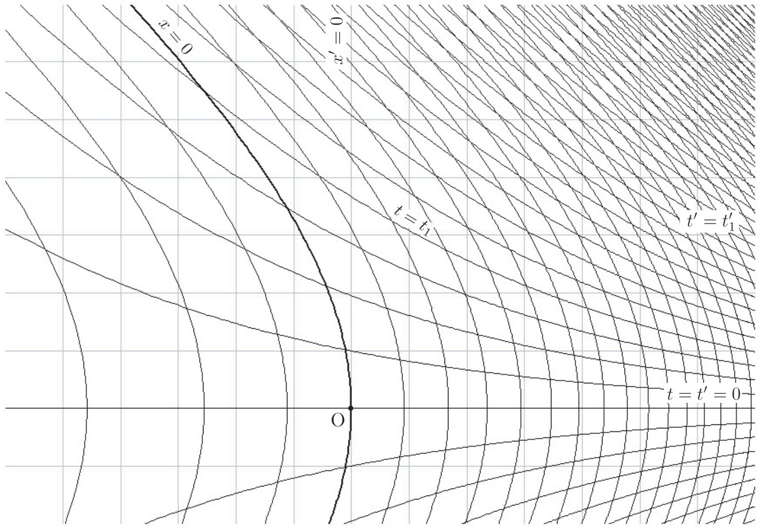

Figure 3: Coordinates frame consisting of the spatial and time lines of S-system (black lines) drawn in the grid of S′-system. The bold line curve is the world line of the spatial point in S-system that was coincident with the spatial origin of S′-system at the time $t = t' = 0$. The point is taken as the spatial coordinate origin of S-system.

Then, similarly to Eqs. (27) and (28), we obtain

$$\begin{aligned} t &= \frac{\tau'_1 + \tau'_{-1}}{2} = \frac{c}{\alpha} e^{\frac{\alpha}{c^2} x'} \sinh \frac{\alpha}{c} t', \\ x &= c \frac{\tau'_1 - \tau'_{-1}}{2} = \frac{c^2}{\alpha} e^{\frac{\alpha}{c^2} x'} \cosh \frac{\alpha}{c} t' - \frac{c^2}{\alpha}, \end{aligned} \tag{46}$$

where $\bar{t}'$ and $\bar{x}'$ are replaced by t' and x', respectively. These are Eq. (13).

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