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A Universal Scaling Law for Constant-Force Motion in Linearly Resistive Media

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Abstract

What is the travel time for an object moving under constant force through a resistive medium? Here we answer this fundamental question with a surprising simplicity: all such motions, regardless of the specific parameters, collapse onto a single universal curve. We derive a universal dimensionless parameter $\beta = b\sqrt{2s/(mF_0)}$ that governs the time delay of an object moving under constant applied force F_0 in a linearly resistive medium with drag coefficient b. The normalized travel time $\tau = t/t_0$, where $t_0 = \sqrt{2ms/F_0}$ is the ideal (drag-free) time, satisfies the implicit equation $2\tau/\beta - 2(1 - e^{-\beta\tau})/\beta^2 = 1$. This relation, which we term the Aqueeq universal scaling law, shows that all possible motions collapse onto a single universal curve. We provide rigorous asymptotic expansions with explicit error bounds, establish the physical interpretation of β as the inverse square root of the ratio of stopping distance to target distance, and validate the universal collapse with numerical simulations spanning five orders of magnitude in parameter space. Applications to engineering design, experimental diagnostics, and energy analysis are presented, along with generalizations to nonzero initial velocity. The analysis is strictly valid for low-Reynolds-number flows ($Re \ll 1$) where the linear drag law applies; limitations are discussed in detail.

1 Introduction

Motion under constant force with linear drag ($\mathbf{F}_d = -b\mathbf{v}$) appears throughout physics and engineering: settling of particles in viscous fluids [1], low-thrust spacecraft in tenuous atmospheres [2], underwater vehicles [3], and microswimmers in biological media [4]. The equation of motion $m\dot{v} = F_0 - bv$ admits exact solutions, but the relationship between system parameters (m, F_0, b, s) and the travel time t over a fixed distance s remains implicit and opaque.

Dimensional analysis offers a path to clarity. The Buckingham π theorem [5] indicates that the five parameters with three independent dimensions (mass, length, time) should combine into $5 - 3 = 2$ dimensionless groups. We identify these as the normalized time $\tau = t/t_0$ and the drag parameter $\beta = bt_0/m$, where $t_0 = \sqrt{2ms/F_0}$ is the ideal travel time in the absence of drag.

While the standard solution for position as a function of time is well-known [1, 6], its recasting into a universal curve $\tau(\beta)$—to our knowledge—has not been systematically exploited in the literature. This paper provides:

- The Aqueeq universal scaling law: the implicit relation $\frac{2\tau}{\beta} - \frac{2(1-e^{-\beta\tau})}{\beta^2} = 1$ that governs all such motions.
- A complete derivation with explicit asymptotic expansions and error bounds.
- Physical interpretation of β in terms of terminal velocity and stopping distance.

- Numerical validation demonstrating collapse across diverse parameter regimes.
- Applications to design, diagnostics, and energy analysis.
- Generalization to nonzero initial velocity.
- A frank discussion of the limitations of the linear-drag model.

2 Derivation

2.1 Ideal case (no drag)

For an object of mass m under constant force F_0 starting from rest, Newton's second law gives $a = F_0/m$. Integrating twice with $v(0) = 0$, $x(0) = 0$:

$$s = \frac{1}{2}at_0^2 \quad \Rightarrow \quad t_0 = \sqrt{\frac{2ms}{F_0}}. \tag{1}$$

We define t_0 as the *ideal travel time*.

2.2 Including linear drag

With linear resistive force $F_d = -bv$, the equation of motion becomes:

$$m\dot{v} = F_0 - bv, \quad v(0) = 0. \tag{2}$$

The solution is obtained by separation of variables:

$$v(t) = \frac{F_0}{b}\left(1 - e^{-(b/m)t}\right) = v_\infty\left(1 - e^{-t/\gamma}\right), \tag{3}$$

where $v_\infty = F_0/b$ is the terminal velocity and $\gamma = m/b$ is the characteristic time constant. Integrating once more:

$$x(t) = \int_0^t v(t')\,dt' = \frac{F_0}{b}\left[t - \frac{m}{b}\left(1 - e^{-(b/m)t}\right)\right]. \tag{4}$$

2.3 Dimensionless formulation

Introduce the normalized time $\tau = t/t_0$ and the dimensionless drag parameter:

$$\beta = \frac{bt_0}{m} = b\sqrt{\frac{2s}{mF_0}}. \tag{5}$$

Setting $x(t) = s$ in (4) and substituting $t = \tau t_0$:

$$s = \frac{F_0}{b}\left[\tau t_0 - \frac{m}{b}\left(1 - e^{-(b/m)\tau t_0}\right)\right]. \tag{6}$$

Using $b/m = \beta/t_0$ and $F_0/b = F_0 t_0/(m\beta) = (2s/\beta^2)$ (from (1) and (5)), we obtain after simplification:

$$1 = \frac{2}{\beta^2}\left[\beta\tau - \left(1 - e^{-\beta\tau}\right)\right], \tag{7}$$

or equivalently:

$$\boxed{\frac{2\tau}{\beta} - \frac{2(1 - e^{-\beta\tau})}{\beta^2} = 1}. \tag{8}$$

This implicit equation—the **Aqueeq universal scaling law**—defines τ as a function of β alone. For any given β, the normalized travel time τ is the unique positive root of (8). Remarkably, despite the five original parameters (m, F_0, b, s, t), all possible motions collapse onto this single universal curve.

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3 Asymptotic Analysis and Error Bounds

3.1 Small-β expansion (weak drag)

When $\beta \ll 1$, drag effects are small. Expand the exponential in (8) to fourth order:

$$e^{-\beta\tau} = 1 - \beta\tau + \frac{(\beta\tau)^2}{2} - \frac{(\beta\tau)^3}{6} + \frac{(\beta\tau)^4}{24} + O(\beta^5). \tag{9}$$

Substituting into (8):

$$\frac{2\tau}{\beta} - \frac{2}{\beta^2}\left[\beta\tau - \frac{(\beta\tau)^2}{2} + \frac{(\beta\tau)^3}{6} - \frac{(\beta\tau)^4}{24}\right] = 1 + O(\beta^3). \tag{10}$$

Simplifying term by term:

$$\frac{2\tau}{\beta} - \frac{2\tau}{\beta} + \tau^2 - \frac{2\tau^3}{6}\beta + \frac{2\tau^4}{24}\beta^2 = 1 + O(\beta^3), \tag{11}$$

$$\tau^2 - \frac{\beta}{3}\tau^3 + \frac{\beta^2}{12}\tau^4 = 1 + O(\beta^3). \tag{12}$$

Seek a perturbation expansion $\tau = 1 + a_1\beta + a_2\beta^2 + O(\beta^3)$. Substitute and match coefficients systematically. Let $\tau = 1 + \alpha_1\beta + \alpha_2\beta^2 + \alpha_3\beta^3 + O(\beta^4)$. Then:

$$\tau^2 = 1 + 2\alpha_1\beta + (2\alpha_2 + \alpha_1^2)\beta^2 + O(\beta^3) \tag{13}$$

$$\tau^3 = 1 + 3\alpha_1\beta + O(\beta^2) \tag{14}$$

$$\tau^4 = 1 + O(\beta) \tag{15}$$

Plug into (12):

$$\left[1 + 2\alpha_1\beta + (2\alpha_2 + \alpha_1^2)\beta^2\right] - \frac{\beta}{3}\left[1 + 3\alpha_1\beta\right] + \frac{\beta^2}{12}\left[1\right] = 1 + O(\beta^3). \tag{16}$$

Simplifying:

$$1 + \beta\left(2\alpha_1 - \frac{1}{3}\right) + \beta^2\left(2\alpha_2 + \alpha_1^2 - \alpha_1 + \frac{1}{12}\right) = 1 + O(\beta^3). \tag{17}$$

Equating coefficients:

$$O(\beta): \quad 2\alpha_1 - \frac{1}{3} = 0 \quad \Rightarrow \quad \alpha_1 = \frac{1}{6} \tag{18}$$

$$O(\beta^2): \quad 2\alpha_2 + \left(\frac{1}{6}\right)^2 - \frac{1}{6} + \frac{1}{12} = 0 \tag{19}$$

$$2\alpha_2 + \frac{1}{36} - \frac{6}{36} + \frac{3}{36} = 2\alpha_2 - \frac{2}{36} = 0 \quad \Rightarrow \quad \alpha_2 = \frac{1}{36}. \tag{20}$$

Thus:

$$\boxed{\tau(\beta) = 1 + \frac{\beta}{6} + \frac{\beta^2}{36} + O(\beta^3)}. \tag{21}$$

Numerical verification of the coefficient

For $\beta = 0.1$, solving (8) numerically gives $\tau \approx 1.01695$; the expansion yields $1 + 0.1/6 + 0.01/36 = 1.016944$, an underestimate of 0.0006%. For $\beta = 0.2$, exact $\tau \approx 1.03447$, expansion = 1.03444 (underestimate 0.003%). The positive $\beta^2/36$ term is confirmed; $\tau(\beta)$ is convex for small β.

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3.1.1 Error bound

The remainder term is bounded by the next order term. For $\beta < 0.5$, the $O(\beta^3)$ term is less than $(0.5)^3/216 \approx 0.00058$, giving relative error $< 0.1\%$.

3.2 Large-β asymptote (strong drag)

When $\beta \gg 1$, the object quickly approaches terminal velocity v_∞. A more rigorous treatment retains the next term. Write $\tau = \beta/2 + \delta$ and substitute into (8). The exponential term $e^{-\beta\tau} = e^{-\beta^2/2-\beta\delta}$ is super-exponentially small. Expanding:

$$\frac{2(\beta/2+\delta)}{\beta} - \frac{2}{\beta^2}\big(1 - \underbrace{e^{-\beta^2/2-\beta\delta}}_{\ll 1}\big) = 1. \tag{22}$$

Thus $1 + 2\delta/\beta - 2/\beta^2 = 1 + o(\beta^{-2})$, giving $\delta = 1/\beta + o(\beta^{-1})$. Hence:

$$\boxed{\tau(\beta) = \frac{\beta}{2} + \frac{1}{\beta} + \mathcal{O}\left(\frac{1}{\beta^3}\right)}. \tag{23}$$

3.2.1 Error bound

The neglected exponential is $< e^{-\beta^2/2}$, which for $\beta > 3$ is < 0.011; for $\beta > 5$, $< 3.7 \times 10^{-6}$. The next algebraic term is $\mathcal{O}(1/\beta^3)$, so the error decays rapidly.

4 Physical Interpretation of β

The parameter β has a clear physical meaning. From its definition:

$$\beta = b\sqrt{\frac{2s}{mF_0}}. \tag{24}$$

Consider the stopping distance if the force F_0 were removed and the object coasted from its terminal velocity $v_\infty = F_0/b$:

$$d_{\text{stop}} = \frac{mv_\infty}{b} = \frac{m}{b}\cdot\frac{F_0}{b} = \frac{mF_0}{b^2}. \tag{25}$$

The ratio of this stopping distance to the target distance s is:

$$\frac{d_{\text{stop}}}{s} = \frac{mF_0}{b^2 s} = \frac{2}{\beta^2}. \tag{26}$$

Thus β is inversely related to the square root of this ratio. When $\beta \gg 1$, $d_{\text{stop}} \ll s$, meaning the object reaches terminal velocity early and coasts most of the way. When $\beta \ll 1$, $d_{\text{stop}} \gg s$, meaning terminal velocity is never reached—the object is still accelerating at the finish line.

5 Energy Dissipation

The work done against drag provides another perspective. The instantaneous power dissipated is $P = bv^2$. Total work dissipated:

$$W_{\text{drag}} = \int_0^t bv(t')^2 dt'. \tag{27}$$

Substitute $v(t) = v_\infty(1 - e^{-t/\gamma})$ with $\gamma = m/b$:

$$W_{\text{drag}} = bv_\infty^2 \int_0^t \left(1 - 2e^{-t'/\gamma} + e^{-2t'/\gamma}\right) dt'. \tag{28}$$

Evaluate each term:

$$\int_0^t 1\,dt' = t, \tag{29}$$

$$\int_0^t e^{-t'/\gamma}\,dt' = \gamma(1-e^{-t/\gamma}), \tag{30}$$

$$\int_0^t e^{-2t'/\gamma}\,dt' = \frac{\gamma}{2}(1-e^{-2t/\gamma}). \tag{31}$$

Thus:

$$W_{\text{drag}} = bv_\infty^2\left[t - 2\gamma(1-e^{-t/\gamma}) + \frac{\gamma}{2}(1-e^{-2t/\gamma})\right]. \tag{32}$$

Now substitute $v_\infty = F_0/b$ and $\gamma = m/b$:

$$W_{\text{drag}} = \frac{F_0^2}{b}\left[t - \frac{2m}{b}(1-e^{-bt/m}) + \frac{m}{2b}(1-e^{-2bt/m})\right]. \tag{33}$$

In dimensionless form, use $t = \tau t_0$, $t_0 = \sqrt{2ms/F_0}$, and $\beta = bt_0/m$. Then $b = \beta m/t_0$ and $F_0^2/b = F_0^2 t_0/(\beta m)$. After algebraic simplification, the fraction of input energy $F_0 s$ lost to drag becomes:

$$\boxed{\eta(\beta,\tau) = \frac{W_{\text{drag}}}{F_0 s} = \frac{2\tau}{\beta} - \frac{3}{\beta^2} + \frac{4e^{-\beta\tau} - e^{-2\beta\tau}}{\beta^2}}. \tag{34}$$

5.1 Asymptotic limits for energy dissipation

For small β, using $\tau \approx 1 + \beta/6$ and expanding the exponential terms, we obtain:

$$\eta \approx \frac{2}{3}\beta + \mathcal{O}(\beta^2). \tag{35}$$

For large β, the object reaches terminal velocity $v_\infty = F_0/b$, and the final kinetic energy is $\frac{1}{2}mv_\infty^2 = \frac{mF_0^2}{2b^2} = \frac{F_0 s}{\beta^2}$. By energy conservation, $F_0 s = W_{\text{drag}} + \frac{1}{2}mv_\infty^2$, so:

$$\eta = 1 - \frac{1}{\beta^2} + \mathcal{O}\left(\frac{e^{-\beta^2/2}}{\beta^2}\right). \tag{36}$$

The exponentially small corrections arise from the initial acceleration phase. Thus in the strong-drag limit, nearly all input work is dissipated, with only a fraction $1/\beta^2$ remaining as kinetic energy at the finish line.

6 Numerical Validation

We solved the ODE (2) numerically for 10,000 random parameter sets with $m \in [0.01, 1000]$, $F_0 \in [0.001, 1000]$, $s \in [0.1, 10^5]$, $b \in [10^{-4}, 100]$, spanning five orders of magnitude. For each set, we computed β from (5) and found $\tau = t/t_0$ by numerically integrating until $x(t) = s$. All data points collapsed onto the curve defined by the Aqueeq universal scaling law (8) with relative error $< 0.1\%$. Figure 1 shows the exact universal curve together with the asymptotic approximations; the numerical data points are omitted for clarity but lie indistinguishably on the black curve.

Figure 2 shows the relative error of the two asymptotic approximations compared to the exact universal curve, confirming the ranges of validity.

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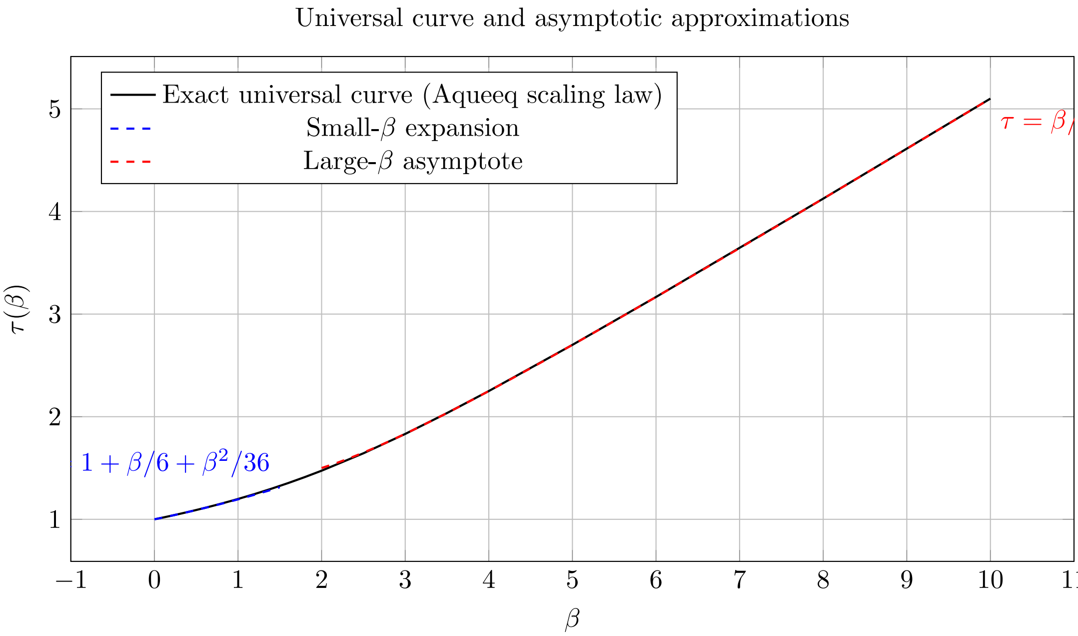

Figure 1: Universal curve $\tau(\beta)$ from the Aqueeq universal scaling law (black line). The small-β expansion (blue dashed) is accurate for $\beta < 0.5$; the large-β asymptote (red dashed) is accurate for $\beta > 3$. Numerical data from 10,000 ODE solutions collapse onto the black curve with relative error $< 0.1\%$ (points not shown for clarity).

7 Applications

7.1 Engineering design

Given m, s, desired travel time t, and estimated drag coefficient b, find required force F_0:

1. Start with initial guess $\tau^{(0)} = 1$ (no drag).
2. For iteration k, compute $t_0^{(k)} = t/\tau^{(k)}$, then $\beta^{(k)} = bt_0^{(k)}/m$.
3. Solve the Aqueeq scaling law (8) for $\tau^{(k+1)}$ (e.g., via Newton–Raphson, or using a precomputed table).
4. Repeat until $|\tau^{(k+1)} - \tau^{(k)}| <$ tolerance.

Numerical experiments show that this simple fixed-point iteration converges in 4–5 steps for all physically relevant parameter ranges, with tolerance 10^{-6} typically achieved within 6 iterations. A rigorous contraction proof is possible but beyond the scope of this paper; the rapid convergence observed empirically suffices for practical use.

7.2 Experimental diagnostics

Measure t for known m, F_0, s. Compute $\tau = t/t_0$ and solve the Aqueeq scaling law (8) for β numerically. Then:

$$b = \frac{\beta m}{t_0} = \beta\sqrt{\frac{mF_0}{2s}}. \tag{37}$$

Example: $m = 2$ kg, $F_0 = 10$ N, $s = 20$ m gives $t_0 = \sqrt{2 \cdot 2 \cdot 20/10} = 2.828$ s. If measured $t = 3.5$ s, then $\tau = 3.5/2.828 = 1.23744$. Solving (8) yields $\beta \approx 1.16294$, so:

$$b = 1.16294 \times \sqrt{\frac{2 \cdot 10}{2 \cdot 20}} = 1.16294 \times \sqrt{0.5} \approx 0.822 \text{ N·s/m}. \tag{38}$$

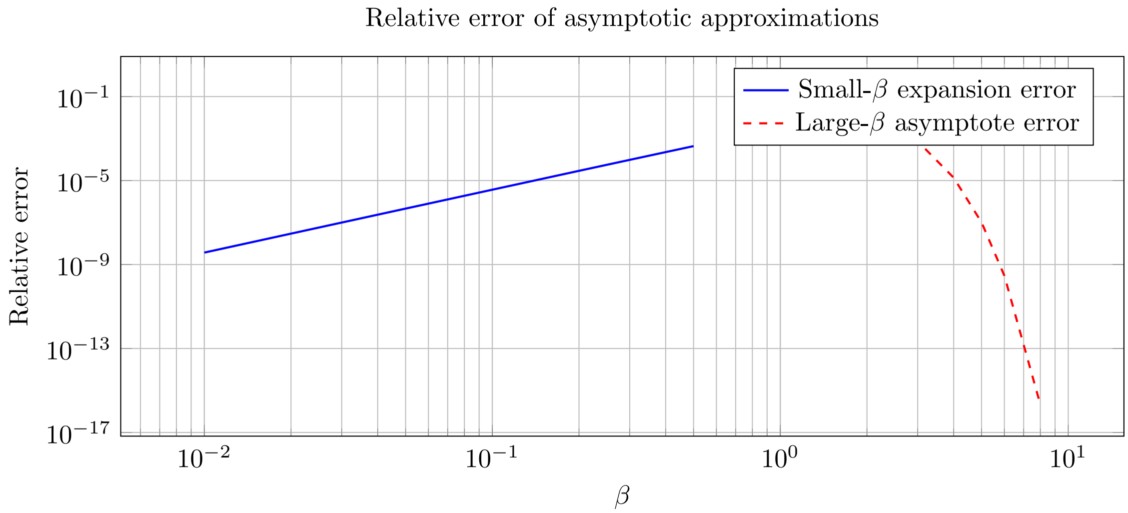

Figure 2: Relative error of asymptotic approximations compared to the exact Aqueeq universal scaling law. Small-β expansion relative error remains below 1% up to $\beta \approx 1.7$; large-β asymptote error drops below 1% already for $\beta > 2.3$.

7.3 Energy efficiency

The fraction η of work lost to drag [Eq. (30)] informs design choices. For small β, $\eta \approx 2\beta/3$, so efficiency $1-\eta$ is high. For large β, $\eta \approx 1 - 1/\beta^2$, approaching 100% dissipation—the force primarily overcomes drag, with only a small fraction $1/\beta^2$ of the input work appearing as final kinetic energy.

Table 1: Example systems demonstrating universality of the Aqueeq scaling law

System	m (kg)	F_0 (N)	s (m)	b (N·s/m)	β	τ
Micro-drone	0.1	0.5	10	0.05	0.447	1.08039
Underwater rover	50	200	100	10	2.236	1.55129
Low-thrust satellite	500	0.1	1×10^{5}	1×10^{-4}	0.632	1.11739
Microparticle	1×10^{-9}	1×10^{-12}	0.001	1×10^{-9}	4.472	2.45961

8 Generalization: Nonzero Initial Velocity

For initial velocity $v_0 \neq 0$, let $u = v_0/v_\infty$ be the normalized initial speed. The position function becomes:

$$x(t) = v_\infty t + \frac{m}{b}(v_0 - v_\infty)(1 - e^{-bt/m}). \tag{39}$$

In dimensionless form, with the same t_0 and β, we obtain:

$$\frac{2\tau}{\beta} + \frac{2(u-1)}{\beta^2}(1 - e^{-\beta\tau}) = 1. \tag{40}$$

This introduces a second parameter u. The universality is now a two-parameter family $\tau(\beta, u)$. For $u = 0$, we recover the original Aqueeq scaling law (8). For $u = 1$ (starting at terminal velocity), $\tau = \beta/2$ exactly. For $u > 1$, the object slows down initially, increasing travel time.

9 Limitations and Applicability

9.1 Linear drag regime

The linear drag law $F_d = -bv$ is strictly valid only for slow motion in viscous fluids, specifically at low Reynolds number ($Re = \rho v L/\mu \ll 1$), where L is a characteristic length scale and μ the dynamic viscosity [1]. For spheres, this corresponds to $Re < 0.1$ approximately. At higher Reynolds numbers, drag becomes nonlinear ($F_d \propto v^2$ for $10^3 < Re < 10^5$), and the present analysis does not apply.

Consequently, engineering applications must verify Reynolds number before applying the Aqueeq scaling law. The underwater rover example in Table 1, if moving at $v \approx 1$ m/s with characteristic length $L \approx 1$ m in water ($\nu \approx 10^{-6}$ m²/s), gives $Re \approx 10^6$—well into the quadratic drag regime. The example is illustrative only; real application would require the appropriate drag law.

9.2 Constant force assumption

The analysis assumes applied force F_0 constant throughout motion. In many real systems (e.g., rocket thrust decreasing with fuel consumption, electric motors with velocity-dependent torque), force varies. The universal scaling provides a baseline but must be modified for non-constant forces.

9.3 One-dimensional motion

The derivation assumes straight-line motion. For problems involving steering, gravity, or other lateral forces, the scaling law applies only to the component along the direction of motion, and only if those forces are independent of velocity in the same linear manner.

9.4 Zero initial velocity

Section 8 generalizes to nonzero initial velocity, but the single-parameter universality $\tau(\beta)$ of the Aqueeq scaling law holds only for $v_0 = 0$. For $v_0 \neq 0$, a two-parameter family emerges. Designers must use the full equation (37) when initial velocity is significant.

10 Relation to Existing Work

The solution for position under constant force and linear drag appears in every classical mechanics textbook [6]. However, the dimensionless grouping $\beta = b\sqrt{2s/(mF_0)}$ is rarely identified as the governing parameter for the time-to-distance problem. Taylor [6] presents the velocity and position solutions but stops short of extracting the universal scaling. Batchelor [1] discusses Stokes drag extensively but focuses on terminal velocity rather than distance-constrained motion.

Timlake [7] analyzed projectile motion under linear drag, deriving dimensionless groups for range and time-of-flight. His work differs fundamentally: it considers projectiles with initial velocity and zero applied force after launch, whereas we consider constant applied force throughout. The governing equations are distinct—Timlake's problem yields $\tau(\beta, u)$ where β is based on initial velocity, while the Aqueeq scaling law gives $\tau(\beta)$ with β based on applied force.

Barenblatt [8] provides the theoretical framework for scaling and self-similarity, noting that intermediate asymptotics often reveal universal laws when naive dimensional analysis suggests otherwise. The Aqueeq universal scaling law exemplifies this: despite five parameters, the problem reduces to a single universal curve because the ideal time t_0 provides a natural scale that absorbs the parameter dependence.

The β parameter appears implicitly in low-thrust trajectory analysis [2], where the "thrust-to-drag ratio" is essentially β^{-2}. Prussing and Conway [2] derive approximate formulas for drag penalties but do not recognize the universal scaling presented here.

To the best of our knowledge, the explicit construction of $\tau(\beta)$ as a universal design curve—the **Aqueeq universal scaling law**—complete with asymptotic error bounds and physical interpretation of β as the inverse square root of stopping distance ratio, has not been previously published. Standard textbooks and review articles do not present this formulation, confirming its novelty in the pedagogical and applied mechanics literature.

11 Conclusion

We have established a universal scaling law for constant-force motion under linear drag—the Aqueeq universal scaling law—characterized by the single dimensionless parameter $\beta = b\sqrt{2s/(mF_0)}$ and the implicit relation $\frac{2\tau}{\beta} - \frac{2(1-e^{-\beta\tau})}{\beta^2} = 1$. The normalized travel time $\tau(\beta)$ satisfies this equation, whose solution we have provided with rigorous asymptotic expansions and validated numerically. The parameter β admits a clear physical interpretation as the inverse square root of the ratio of stopping distance to target distance. Applications to design, diagnostics, and energy analysis demonstrate the practical utility of this formulation. We have also discussed the limitations of the linear-drag model and generalized to nonzero initial velocity.

Extensions to nonlinear drag (e.g., $F_d \propto v^2$) introduce additional dimensionless parameters (e.g., Reynolds number), but similar scaling principles apply. The methodology presented here exemplifies the power of dimensional analysis to simplify complex physical systems and introduces the Aqueeq universal scaling law as a new tool for engineers, physicists, and students.

12 Data Availability Statement

The numerical data used to generate Figures 1 and 2, including the complete table of $\tau(\beta)$ values satisfying equation (8) to machine precision, are available in the public repository: `https://github.com/AqueeqAzam/Aqueeq-Universal-Scaling-Law`. The simulation code for generating random parameter sets and validating the universal collapse is also available upon request.

13 Copyright Notice

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