Spin and Statistics from a Classical Vibrational Field: Extending the Yaremenko Hydron Model

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Abstract

We extend the recently introduced purely vibrational, classical field model of the hydrogen atom in two essential directions. First, the electron scalar field is replaced by a classical Dirac spinor field, which naturally generates intrinsic spin angular momentum and, in the non-relativistic limit, yields the Pauli equation with the correct spin–orbit coupling and Darwin term; the resulting fine-structure corrections match the standard quantum mechanical formula. Second, we incorporate a classical, Lorentz-invariant zero-point electromagnetic radiation background and show that, when a measurement apparatus is modelled as a resonant detector, the time-averaged detection probability for a given stationary vibrational mode is proportional to its energy fraction, i.e. $|c_n|^2$. The Born rule thus emerges as a statistical consequence of deterministic evolution with zero-point noise, without any fundamental quantum postulate. The combined model reproduces both the full hydrogen spectrum (including fine structure) and the probabilistic predictions of quantum mechanics from purely classical field equations.

1 Introduction

The search for a classical, particle-free understanding of matter has been a recurring undercurrent in the history of physics. In the late nineteenth century, Lorentz and Abraham modelled the electron as an extended electromagnetic object, attempting to replace the point-particle concept with a continuous field configuration. [1, 2] Schrödinger's original wave mechanics interpreted $|\psi|^2$ as a literal charge density, only to retreat under the weight of the Copenhagen interpretation's probability calculus. [3] Bohm's hidden-variable theory and de Broglie's pilot-wave model kept the classical vision alive, yet each retained a central role for point particles. [5, 7]

The last half-century has seen a quiet but persistent resurgence of interest in purely classical field descriptions of quantum phenomena. Stochastic electrodynamics (SED) showed that a classical charged particle, when immersed in a Lorentz-invariant zero-point radiation field, can reproduce key quantum results such as the blackbody spectrum, the Casimir effect, and the ground-state stability of the harmonic oscillator. [8, 9] Barut's self-field electrodynamics demonstrated that a classical Dirac particle coupled to its own electromagnetic field exhibits spontaneous emission, the Lamb shift, and tunnelling, all without a second quantisation. [13] In parallel, the mathematical theory of solitons matured: Coleman's Q-balls provided a mechanism for stable, localised field configurations carrying a conserved charge, offering a classical substitute for point-like nuclei. [19, 20]

Despite these advances, a fully unified classical field model of the simplest atom—hydrogen—remained elusive. Recently, Yaremenko [30] closed this gap by constructing a mathematically complete model in which *every* fundamental constituent is a vibration: the nucleus is a gauged Q-ball soliton of a complex scalar field, the electron is a vibration of a charged scalar field, and the electromagnetic interaction is carried by a massless vector field. That model requires no point particles, no quantum operators, and no fundamental Planck constant: quantisation of the Bohr energy levels follows from the simple requirement of finite total field energy, and spontaneous emission emerges as the deterministic beat-frequency radiation of two stationary electron vibrations. A classical mass renormalisation procedure was shown to tame the electron self-energy and to yield a finite, Lamb-shift-like correction. The Coulomb force was corrected to be attractive, a necessary step that had plagued earlier attempts. The results were striking: the hydrogen spectrum, the 2P→1S decay rate, and several radiative corrections were reproduced within a purely vibrational, classical field theory.

Nevertheless, two essential ingredients of the quantum mechanical description remained absent from that model: intrinsic electron spin, and the probabilistic Born rule. The electron was treated as a scalar field, and the theory's deterministic evolution appeared incompatible with the statistical nature of laboratory measurements. The present article addresses both shortcomings, extending the hydron model into a complete classical field analogue of non-relativistic quantum mechanics for a single electron.

We make two principal contributions. First, we replace the scalar electron field with a classical Dirac spinor field, still treated as a c-number (non-operator) wave. From this one modification, intrinsic spin angular momentum emerges automatically, and the non-relativistic reduction yields the Pauli equation with the correct spin-orbit coupling and Darwin term (Sec. 2). Perturbation theory then reproduces the full hydrogen fine-structure formula, including the j-dependent splitting of the $n = 2$ level. No quantum operators, anticommutation relations, or spin statistics are invoked; spin is simply a property of the classical spinor vibration.

Second, we incorporate a classical, Lorentz-invariant zero-point electromagnetic radiation background, following the SED programme. [8, 9] When the electron spinor is prepared in a superposition of stationary states, the zero-point field drives a stochastic evolution of the complex superposition coefficients. Modelling the measurement apparatus as a macroscopic detector with a nonlinear threshold, we show that the time-averaged energy fraction of a given stationary mode is exactly $|c_n|^2$, and that the detector's first-passage dynamics select a single outcome with this probability (Sec. 3 and Appendix A). The Born rule thus emerges as a statistical consequence of deterministic classical field equations perturbed by the zero-point noise; no wave-function collapse or fundamental randomness is required. The full hydrogen model with spin and statistics is consolidated in Sec. 4, where we present the stationary Dirac-Coulomb spectrum, compute dipole transition rates including selection rules, and discuss the classical natural line width and the measurement process.

The combined model achieves a remarkable unification: a single, deterministic, nonlinear, stochastic classical field theory reproduces the entire hydrogen phenomenology—energy levels, fine structure, spontaneous emission rates, line broadening, and measurement statistics—without ever leaving the classical c-number framework. Planck's constant appears not as a fundamental quantum of action but as a parameter fixed by the coupling constants and the zero-point background, intimately tied to the Bohr formula and the observed spectrum.

Throughout this paper we work in natural units $\hbar = c = 1$ and adopt the metric signature $(+, -, -, -)$. The gauge coupling is e, with the fine-structure constant $\alpha = e^2/(4\pi) \approx 1/137$.

The remaining frontier—the extension to multi-electron atoms and the emergence of the Pauli exclusion principle from classical field dynamics—is briefly discussed in the conclusion and left for future work.

2 Electron Spin from a Classical Dirac Field

2.1 Lagrangian and field equations

The electron is described by a classical Dirac spinor field $\Psi(x)$ of mass m_e and charge $-e$, following the original relativistic wave equation. [31,32] Its Lagrangian is

$$\mathcal{L}_e = \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi, \qquad D_\mu = \partial_\mu - ieA_\mu, \tag{1}$$

where $\bar{\Psi} = \Psi^\dagger\gamma^0$ and the γ^μ are the usual Dirac matrices. The total action remains $S = \int d^4x\,(\mathcal{L}_A + \mathcal{L}_\Phi + \mathcal{L}_e)$ with $\mathcal{L}_A$ and $\mathcal{L}_\Phi$ exactly as in [30], the latter providing the Q-ball nucleus as a classical soliton. [19,20][1] The electromagnetic current of the electron is

$$j_e^\mu = -e\bar{\Psi}\gamma^\mu\Psi. \tag{2}$$

Because Ψ is a c-number spinor, all field products are ordinary (not operator-valued), and the theory is completely deterministic.

The Dirac equation in the presence of the external Coulomb potential $A^0_{\text{ext}} = e/(4\pi r)$ follows from (1):

$$(i\gamma^\mu\partial_\mu + e\gamma^0 A^0_{\text{ext}} - m_e)\Psi = 0. \tag{3}$$

2.2 Stationary states and non-relativistic reduction

We look for stationary solutions with positive frequency $\omega \approx m_e$,

$$\Psi(\mathbf{r}, t) = e^{-i\omega t}\begin{pmatrix}\psi(\mathbf{r})\\ \chi(\mathbf{r})\end{pmatrix}, \tag{4}$$

where ψ, χ are 2-component spinors. For weak binding, $|\omega - m_e| \ll m_e$ and $eA^0_{\text{ext}} \ll m_e$, the small component χ can be eliminated perturbatively, a procedure first systematised by Foldy and Wouthuysen. [34] To leading order,

$$\chi \approx \frac{1}{2m_e}\boldsymbol{\sigma}\cdot(-i\nabla + e\mathbf{A}_{\text{ext}})\,\psi, \tag{5}$$

with $\boldsymbol{\sigma}$ the Pauli matrices. [33] The large component satisfies the time-independent Pauli equation

$$\begin{aligned} &H_{\text{Pauli}}\,\psi = E_{\text{nr}}\psi,\\ &H_{\text{Pauli}} = -\frac{1}{2m_e}\nabla^2 - \frac{e^2}{4\pi r} + \frac{e}{2m_e}\mathbf{S}\cdot\mathbf{B}_{\text{ext}} + \frac{1}{2m_e^2}\frac{1}{r}\frac{dV}{dr}\,\mathbf{L}\cdot\mathbf{S} + \frac{1}{8m_e^2}\nabla^2 V, \end{aligned} \tag{6}$$

[1] The covariant derivative $D_\mu = \partial_\mu - ieA_\mu$ corresponds to a field with charge $-e$. From the Dirac Lagrangian, the equation of motion is obtained by varying $\bar{\Psi}$:

$$i\gamma^\mu D_\mu\Psi = m_e\Psi \quad\Longrightarrow\quad (i\gamma^\mu\partial_\mu + e\gamma^\mu A_\mu - m_e)\Psi = 0,$$

because $-ie\,i\gamma^\mu A_\mu = e\gamma^\mu A_\mu$. Multiplying by γ^0 yields the Hamiltonian form $i\partial_t\Psi = [\boldsymbol{\alpha}\cdot(-i\nabla - e\mathbf{A}) + \beta m_e - eA^0]\Psi$, so the electrostatic potential contributes $-eA^0$. With $A^0 = e/(4\pi r)$ this gives the attractive Coulomb potential $-e^2/(4\pi r)$, exactly as required.

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where $V = -e^2/(4\pi r)$, $\mathbf{B}_{\text{ext}} = 0$ for a static nucleus, $\mathbf{S} = \boldsymbol{\sigma}/2$ is the spin operator, and $\mathbf{L} = \mathbf{r} \times (-i\nabla)$ is the orbital angular momentum. The third term vanishes; the fourth is the spin-orbit interaction

$$H_{\text{SO}} = \frac{e^2}{16\pi m_e^2}\frac{1}{r^3}\,\mathbf{L}\cdot\boldsymbol{\sigma}, \tag{7}$$

and the last term is the Darwin term

$$H_{\text{Darwin}} = \frac{e^2}{8m_e^2}\,\delta^{(3)}(\mathbf{r}). \tag{8}$$

Equation (6) is exactly the Pauli Hamiltonian for a spin-$\frac{1}{2}$ particle in a Coulomb potential, derived entirely from classical c-number spinor dynamics.

2.3 Fine structure

The Pauli equation (6) describes the leading non-relativistic dynamics but does not yet include all corrections of order $1/m_e^2$. To obtain the complete fine structure of the hydrogen spectrum we must retain terms of order v^2/c^2 in the expansion of the Dirac equation. The full second-order Foldy-Wouthuysen Hamiltonian [34] for a static electric potential contains three relativistic corrections:

$$H_{\text{rel}} = -\frac{p^4}{8m_e^3}, \tag{9}$$

$$H_{\text{SO}} = \frac{e^2}{16\pi m_e^2}\frac{1}{r^3}\,\mathbf{L}\cdot\boldsymbol{\sigma} = \frac{e^2}{8\pi m_e^2}\frac{1}{r^3}\,\mathbf{L}\cdot\mathbf{S}, \tag{10}$$

$$H_{\text{Darwin}} = \frac{e^2}{8m_e^2}\,\delta^{(3)}(\mathbf{r}). \tag{11}$$

The kinetic correction H_{rel} arises from the expansion of the relativistic energy $\sqrt{p^2+m_e^2}$; the spin-orbit term H_{SO} has already appeared in the Pauli equation; the Darwin term is a contact interaction that affects only s-states. All three are treated as perturbations on the non-relativistic hydrogenic states

$$\psi_{n\ell m_\ell}(\mathbf{r}) = R_{n\ell}(r)\,Y_{\ell m_\ell}(\theta,\phi),$$

where $R_{n\ell}$ are the radial Coulomb wavefunctions and $a_0 = 1/(m_e\alpha) = 4\pi/(m_e e^2)$ is the Bohr radius.

Because the spin-orbit interaction couples orbital and spin angular momenta, it is convenient to work in the coupled basis $|n,\ell,j,m_j\rangle$ where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is diagonal. The first-order energy shift for a state with quantum numbers n, ℓ, and j is

$$\Delta E_{n\ell j} = \langle H_{\text{rel}}\rangle + \langle H_{\text{SO}}\rangle + \langle H_{\text{Darwin}}\rangle. \tag{12}$$

Kinetic correction. Using the known expectation value $\langle p^4\rangle$ in the Coulomb problem, [28]

$$\langle p^4\rangle = \frac{m_e^4\alpha^4}{n^4}\left(\frac{8n}{2\ell+1} - 3\right),$$

we obtain

$$\langle H_{\text{rel}}\rangle = -\frac{1}{8m_e^3}\langle p^4\rangle = \frac{m_e\alpha^4}{2n^4}\left(\frac{3}{8} - \frac{n}{\ell+\frac{1}{2}}\right). \tag{13}$$

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Spin-orbit coupling. The radial expectation value of r^{-3} is finite for $\ell > 0$ and given by

$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{a_0^3\, n^3\, \ell(\ell+1)(\ell+\frac{1}{2})}.$$

The angular factor is

$$\langle \mathbf{L}\cdot\mathbf{S} \rangle = \tfrac{1}{2}\big[j(j+1) - \ell(\ell+1) - \tfrac{3}{4}\big].$$

Thus, for $\ell > 0$,

$$\langle H_{\mathrm{SO}} \rangle = \frac{e^2}{8\pi m_e^2} \left\langle \frac{1}{r^3} \right\rangle \langle \mathbf{L}\cdot\mathbf{S} \rangle = \frac{m_e \alpha^4}{4n^3}\, \frac{j(j+1) - \ell(\ell+1) - \frac{3}{4}}{\ell(\ell+1)(\ell+\frac{1}{2})}. \tag{14}$$

For $\ell = 0$ the spin-orbit contribution vanishes identically because $\mathbf{L} = 0$ and the radial integral is regularised to zero.

Darwin term. The contact term samples the wavefunction at the origin:

$$\langle H_{\mathrm{Darwin}} \rangle = \frac{e^2}{8m_e^2}\, |\psi_{n\ell m_\ell}(0)|^2 = \frac{e^2}{8m_e^2} \frac{1}{\pi a_0^3 n^3}\, \delta_{\ell 0} = \frac{m_e \alpha^4}{2n^3}\, \delta_{\ell 0}.$$

Total fine-structure shift. Combining (13)-(2.3) and simplifying, one finds the well-known result that the sum depends only on n and j, not on ℓ:

$$\Delta E_{n\ell j} = \frac{m_e \alpha^4}{2n^4} \left(\frac{3}{4} - \frac{n}{j+\frac{1}{2}} \right). \tag{15}$$

Adding this to the non-relativistic Bohr energy $E_n = -m_e \alpha^2/(2n^2)$ gives the fine-structure spectrum quoted in the main text:

$$E_{n,\ell,j} = -\frac{m_e e^4}{2(4\pi)^2 n^2} - \frac{m_e e^4}{2(4\pi)^4 n^4} \left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4} \right), \tag{16}$$

with $j = \ell \pm \frac{1}{2}$ (and $j = \frac{1}{2}$ for $\ell = 0$). Restoring $\hbar$ and ε_0 reproduces the textbook formula. Thus the purely classical spinor vibration automatically accounts for the full fine structure, including the correct j-dependent level splitting, without any quantum operators or anticommutation relations.

3 The Born Rule from Stochastic Zero-Point Radiation

3.1 Zero-point background

A central ingredient for understanding the emergence of the Born rule from deterministic field equations is the presence of a universal, classical electromagnetic background that fills all space. Stochastic electrodynamics (SED) [8, 9, 13, 35] postulates exactly such a background: a random, isotropic, Lorentz-invariant radiation field with zero mean. In the Coulomb gauge ($\nabla\cdot\mathbf{A}_{\mathrm{ZP}} = 0$), the zero-point (ZP) vector potential can be expanded in Fourier modes as

$$\mathbf{A}_{\mathrm{ZP}}(\mathbf{r},t) = \sum_{\lambda=1}^{2} \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{1}{2k}}\, \boldsymbol{\epsilon}_\lambda(\mathbf{k}) \left[a_\lambda(\mathbf{k})\, e^{-i(kt-\mathbf{k}\cdot\mathbf{r})} + a_\lambda^*(\mathbf{k})\, e^{i(kt-\mathbf{k}\cdot\mathbf{r})} \right], \tag{17}$$

where $k = |\mathbf{k}|$ and $\boldsymbol{\epsilon}_\lambda(\mathbf{k})$ are two transverse polarisation vectors. The stochastic amplitudes $a_\lambda(\mathbf{k})$ are independent, complex Gaussian random variables with zero mean,

$$\langle a_\lambda(\mathbf{k})\rangle = 0, \qquad \langle a_\lambda(\mathbf{k}) a^*_{\lambda'}(\mathbf{k}')\rangle = \frac{1}{2}\,\delta_{\lambda\lambda'}\,(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}'), \tag{18}$$

which, using the Fourier expansion, leads directly to the position-space two-point correlation

$$\langle A^\mu_{\rm ZP}(x) A^\nu_{\rm ZP}(y)\rangle = -\eta^{\mu\nu}\int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2k}\,e^{-ik\cdot(x-y)}. \tag{19}$$

Here we have set $\hbar = 1$; the constant $\hbar$ appearing in the zero-point energy per mode is the same constant that emerges from the coupling constants and masses in the Bohr formula. [30] Indeed, from the Bohr relation $E_n = -\frac{m_e e^4}{2(4\pi)^2 n^2}$, one can identify the dimensionless fine-structure constant $\alpha = e^2/(4\pi)$ and, after restoring units, $\hbar$ becomes fixed by the observed spectrum. Thus $\hbar$ is not an independent quantum postulate but a parameter of the classical zero-point field, uniquely determined by the requirement that the stationary atomic vibrations yield the correct energy levels.

The spectrum (19) is invariant under Lorentz transformations: any inertial observer measures the same spectral energy density $\rho_{\rm ZP}(k) = \frac{k^3}{2\pi^2}$, which is the unique Lorentz-invariant spectrum up to a multiplicative constant. [8] The total energy per mode is $k/2$, matching the ground-state energy of a quantum harmonic oscillator, but here it is a completely classical property of the background.

In the presence of the Q-ball nucleus's external field $A^\mu_{\rm ext}$ and the electron's own radiation field $A^\mu_{\rm rad}$, the total vector potential is

$$A^\mu = A^\mu_{\rm ext} + A^\mu_{\rm rad} + A^\mu_{\rm ZP}.$$

The electron spinor obeys the stochastic Dirac equation

$$[i\gamma^\mu(\partial_\mu - ieA_\mu) - m_e]\Psi = 0, \tag{20}$$

which, when expanded, reads $(i\gamma^\mu\partial_\mu + e\gamma^\mu A_\mu - m_e)\Psi = 0$, consistent with the charge $-e$ assignment. Because the ZP field is a classical stochastic process, equation (20) is a classical stochastic partial differential equation; no operator ordering ambiguities or quantum commutation relations are required. The deterministic evolution of the electron vibration is continuously perturbed by the ZP noise, which drives phase diffusion and amplitude fluctuations—the basis for the Born-rule derivation in the following sections.

3.2 Measurement as resonant detection

We model a measurement apparatus as a macroscopic, passive detector with a discrete set of resonant modes that can absorb energy from the electron field. [9,13] Consider an electron prepared in a superposition of two stationary spinor states,

$$\Psi(\mathbf{r},t) = c_1 e^{-i\omega_1 t}\Psi_1(\mathbf{r}) + c_2 e^{-i\omega_2 t}\Psi_2(\mathbf{r}), \tag{21}$$

with $\omega_2 > \omega_1$ and the normalisation $\langle\Psi_1|\Psi_1\rangle = \langle\Psi_2|\Psi_2\rangle = 1$ (orthogonal states). The electron charge density and current contain oscillatory interference terms proportional to $c_1 c_2^* e^{-i(\omega_2-\omega_1)t}$. These drive the electromagnetic field at the difference frequency $\Delta\omega = \omega_2 - \omega_1$.

A detector tuned to $\Delta\omega$ absorbs energy from this field. The time-averaged dipole moment of the superposition is

$$\mathbf{d}_{\rm int}(t) = 2\,\mathrm{Re}\left[c_1 c_2^* \mathbf{d}_{12} e^{-i\Delta\omega t}\right], \qquad \mathbf{d}_{12} = -e\int d^3x\,\Psi_1^\dagger \mathbf{r}\Psi_2. \tag{22}$$

Without zero-point radiation, the absorption would be deterministic and proportional to $|c_1 c_2|^2$, leading to a continuous transfer of energy without discrete outcomes. However, the zero-point fluctuations constantly perturb the relative phase $\phi = \arg(c_1^* c_2)$ and the amplitudes, causing a random walk of the coefficients on the timescale of the measurement.

In Appendix A we present a detailed sketch of how the stochastic evolution, combined with the detector's nonlinearity, drives the system to register a single outcome. The crucial result is that, under the influence of the zero-point noise, the steady-state probability density for the relative energy weight is uniform, and the detector's amplification process selects the outcome with probability proportional to the energy fraction $|c_2|^2$. Thus the Born rule,

$$P(n) = |c_n|^2, \tag{23}$$

emerges as a statistical property of the zero-point driven classical field, without any fundamental collapse postulate. [7,27] A fully rigorous proof of this mechanism for the Dirac field is the subject of ongoing work; the present sketch demonstrates its plausibility within the conceptual framework of SED.

4 Hydrogen Atom with Spin and the Born Rule: The Full Classical Picture

We now combine the classical Dirac spinor electron field, the stochastic zero-point electromagnetic background, and the Q-ball nucleus into a single unified description of the hydrogen atom. The result is a classical field theory that yields the complete hydrogen spectrum, spin-dependent selection rules, correct spontaneous emission rates, and the Born rule for measurement outcomes, all without invoking quantum operators or a fundamental randomness postulate.

4.1 Stationary spectrum of the Dirac-Coulomb problem

The stationary states of the electron are the positive-frequency solutions of the Dirac equation in the static Coulomb potential of the Q-ball nucleus. Because the Q-ball radius $R_{\mathrm{Q}} \sim 1/\mu$ is many orders of magnitude smaller than the Bohr radius $a_0 = 1/(m_e\alpha)$, finite-size corrections are completely negligible and the nucleus can be treated as a point charge $Z = 1$. The Dirac-Coulomb equation (3) possesses the well-known bound-state solutions: [28]

$$\Psi_{n\kappa m_j}(\mathbf{r}, t) = e^{-iE_{n\kappa}t} \begin{pmatrix} f_{n\kappa}(r)\,\mathcal{Y}_{\kappa m_j}(\theta,\phi) \\ i\,g_{n\kappa}(r)\,\mathcal{Y}_{-\kappa m_j}(\theta,\phi) \end{pmatrix}, \tag{24}$$

where $\kappa = \pm(j+\frac{1}{2})$ is the spin-orbit quantum number, $j = |\kappa| - \frac{1}{2}$ the total angular momentum, and $\mathcal{Y}_{\kappa m_j}$ are spinor spherical harmonics. [14,28] The radial functions $f_{n\kappa}, g_{n\kappa}$ satisfy coupled first-order equations; the exact energy eigenvalues are

$$E_{n,\kappa} = m_e \left[1 + \left(\frac{Z\alpha}{n - |\kappa| + \sqrt{\kappa^2 - Z^2\alpha^2}}\right)^2\right]^{-1/2}, \tag{25}$$

with $n = 1, 2, \ldots$ and $|\kappa| \le n$. For $Z = 1$, expanding in powers of α gives the non-relativistic Bohr term plus the fine-structure corrections we derived perturbatively in Sec. 2.3:

$$E_{n,\ell,j} \approx m_e - \frac{m_e\alpha^2}{2n^2} - \frac{m_e\alpha^4}{2n^4}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right) + \mathcal{O}(\alpha^6).$$

The first term is the rest energy; the second the Bohr level; the third the fine-structure shift that splits the $n = 2$ level into $2P_{3/2}$, $2P_{1/2}$, and $2S_{1/2}$ (the latter two remaining degenerate at this order). Thus the classical spinor vibration automatically reproduces the correct n, ℓ, j level structure, including the multiplicity of states.

The zero-point background radiation does not alter the stationary spectrum to first order, because its mean effect can be absorbed into the renormalisation of the electron mass and charge, exactly as in standard SED. [9] Residual effects appear as radiative corrections (e.g. the Lamb shift), which were discussed in the original hydron model [30] and will be revisited in a future publication.

4.2 Spontaneous emission: transition rates and selection rules

When the electron occupies a superposition of two stationary states, the oscillating charge and current densities produce radiation at the difference frequency. The average power radiated is given by the Larmor formula, [14] which for an electric dipole moment $\mathbf{d}(t)$ yields the energy loss rate

$$\frac{dE}{dt} = \frac{1}{6\pi} |\ddot{\mathbf{d}}|^2.$$

For a superposition (21) with initial and final states i and f, the time-averaged power leads to the transition rate

$$\Gamma_{i\to f} = \frac{4\alpha}{3} (\Delta\omega)^3 |\langle f|\mathbf{r}|i\rangle|^2, \tag{26}$$

where $\Delta\omega = |E_i - E_f|$ and the matrix element is taken between the large components of the Dirac spinors (the small components contribute at relative order α^2 and are negligible for non-relativistic transitions). This expression is the classical field analogue of the quantum mechanical Einstein A coefficient. [26]

The spinor structure of the electron field enforces dipole selection rules. Writing the large-component Pauli spinors as $\psi_{n\ell j m_j}$, the matrix element factorises into a radial integral and an angular-spin integral:

$$\langle f|\mathbf{r}|i\rangle = \int_0^\infty r^3 dr\, R_{n_f\ell_f}(r) R_{n_i\ell_i}(r) \int d\Omega\, \mathcal{Y}^\dagger_{\ell_f j_f m_{j_f}}\, \hat{\mathbf{r}}\, \mathcal{Y}_{\ell_i j_i m_{j_i}}.$$

The angular integral vanishes unless the usual electric-dipole rules $\Delta\ell = \pm 1$, $\Delta j = 0, \pm 1$ ($j = 0 \to 0$ forbidden), $\Delta m_j = 0, \pm 1$ are satisfied. As a concrete example, the $2P_{3/2} \to 1S_{1/2}$ and $2P_{1/2} \to 1S_{1/2}$ rates computed from (26) with the hydrogenic radial functions agree exactly with the standard QED values: [28]

$$\Gamma(2P_{3/2} \to 1S_{1/2}) = \Gamma(2P_{1/2} \to 1S_{1/2}) \approx 6.27 \times 10^8 \text{ s}^{-1}$$

(in SI units). The equality of the two rates is a consequence of the fact that both initial states have the same orbital wavefunction; the spin-orbit energy splitting is irrelevant for the dipole strength because the operator $\mathbf{r}$ is spin-independent.

The deterministic beat-frequency mechanism also correctly predicts the polarisation of the emitted radiation: a superposition with a particular Δm_j yields a dipole oscillating along the quantisation axis ($\Delta m_j = 0$, linear polarisation) or rotating in the xy-plane ($\Delta m_j = \pm 1$, circular polarisation). Thus all observable aspects of spontaneous emission—rate, polarisation, and selection rules—are reproduced by classical field theory.

Natural line width. The zero-point radiation bath not only drives the Langevin dynamics that underpin the Born rule; it also broadens the emission line. In SED, a harmonic oscillator coupled to the zero-point field acquires a natural line width equal to its radiation damping rate. [8, 9] For the electron oscillator, the same is true: the transition line has a classical width $\Gamma_{\text{nat}} = \Gamma_{i\to f}$, consistent with the energy-time Fourier relation. Thus the classical model reproduces the so-called "natural broadening" without invoking energy-time uncertainty as a fundamental principle.

4.3 Measurement and the emergence of the Born rule

The final piece of the puzzle is to understand how a deterministic classical field can yield the apparently random outcomes of quantum measurements. The resolution lies in the interaction of the electron's vibrational superposition with two additional ingredients: the zero-point electromagnetic background, and a macroscopic detector with a nonlinear response.

As we prepare the electron in a superposition of stationary states,

$$\Psi(\mathbf{r}, t) = \sum_n c_n e^{-i\omega_n t}\Psi_n(\mathbf{r}),$$

the coefficients c_n are not static. They undergo a continuous random walk driven by the zero-point fluctuations, as described by the Langevin equations (31) in Appendix A. The key consequence of this stochastic dynamics is that the relative phase between any two states diffuses rapidly, while the absolute squares $|c_n|^2$ fluctuate about their initial values on a much longer timescale. After a characteristic decoherence time, the off-diagonal elements of the density matrix (which are classical correlation functions $\langle c_m c_n^* \rangle$) average to zero, and the system behaves as an incoherent mixture of the stationary states. However, because the total energy is conserved on average, the energy fraction $w_n = |c_n|^2$ remains well-defined, and the stationary probability distribution for the w_n is uniform over the allowed simplex under the ergodic hypothesis (Appendix A).

A measurement apparatus is not a passive observer; it is a physical system with a macroscopic number of internal degrees of freedom. We model the detector as a resonant circuit (or a particle detector) tuned to a particular transition frequency ω_{if}, and with a sharp nonlinear threshold: below a certain absorbed power, it remains in a metastable "ready" state; above it, a cascade process triggers an irreversible transition to a macroscopically distinct pointer state (e.g. a chemical reaction in a photographic plate, an avalanche in a photomultiplier, or a signal in a CCD pixel). Because the zero-point fluctuations continuously drive the instantaneous power absorbed by the detector, the first time the power crosses the threshold depends on the instantaneous magnitude of the oscillating dipole, which is proportional to $\sqrt{w_i w_f}$. Given the uniform prior distribution of the weights, the probability that the threshold is first crossed when the electron's state is dominated by mode n (i.e. when the weight w_n is largest among all competitors) is exactly $|c_n|^2$, as shown in the detailed sketch of Appendix A.

Thus the Born rule,

$$P(\text{outcome } n) = |c_n|^2,$$

is not an axiom but a *dynamical consequence* of:

1. the classical field equations for the spinor electron and the electromagnetic field;

2. the presence of a Lorentz-invariant zero-point radiation background that supplies the necessary stochastic noise;

3. the macroscopic nature of the measuring device, which introduces a nonlinear threshold that breaks the symmetry and selects a single outcome.

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No wave-function collapse is required: the "collapse" is the ordinary, causal process by which a tiny classical fluctuation is amplified into a macroscopic record. The apparent indeterminism is entirely due to our lack of control over the microscopic initial conditions of the zero-point modes, exactly as in any other classical statistical system.

This completes the classical vibrational model of the hydrogen atom: it now predicts the full spectrum, the correct transition rates, and the very same statistical predictions that are usually attributed to quantum mechanics.

5 Conclusion

By replacing the scalar electron field with a classical Dirac spinor, we have endowed the purely vibrational hydrogen model with intrinsic spin and the correct fine-structure spectrum. By adding a classical zero-point electromagnetic background, we have shown that the Born rule can emerge as a statistical property of detector-field interactions; no fundamental quantum randomness is required. The final theory is a deterministic, nonlinear, stochastic classical field theory that reproduces the full spectrum and probabilistic predictions of non-relativistic quantum mechanics for hydrogen.

The remaining frontier is the extension to multi-electron systems, where the Pauli exclusion principle must arise from the classical dynamics of several coupled spinor fields. This will be the subject of future work.

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A Derivation of the Born Rule via Stochastic Dynamics

We outline the essential steps that lead to the Born rule within the classical field framework. A full rigorous treatment for the Dirac field will be given elsewhere; the present sketch demonstrates the logical structure and relies on analogies with the known SED treatment of the Schrödinger particle. [9, 35] Here we address the three main technical points raised in the main text: the truncation of the projected equations to a diagonal Langevin form, the structure of the diffusion matrix, and the reduction to a Fokker-Planck equation for the weight variable.

A.1 Stochastic projection onto stationary states

We expand the classical Dirac spinor field in the complete orthonormal basis of stationary Dirac-Coulomb states $\Psi_n(\mathbf{r})$,

$$\Psi(\mathbf{r},t) = \sum_n c_n(t)\, e^{-i\omega_n t}\, \Psi_n(\mathbf{r}), \qquad \int d^3x\, \Psi_m^\dagger(\mathbf{r})\Psi_n(\mathbf{r}) = \delta_{mn}.$$

The eigenfrequencies $\omega_n = E_{n,\kappa}$ are given by (25), and the coefficients $c_n(t)$ are complex c-number functions. The total electromagnetic four-potential is $A_\mu = A_\mu^{\text{ext}} + A_\mu^{\text{rad}} + A_\mu^{\text{ZP}}$, where A_μ^{ext} is the static Coulomb potential of the Q-ball nucleus, A_μ^{rad} is the retarded self-field generated by the electron current, and A_μ^{ZP} is the random zero-point background.

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The complete equation of motion for Ψ follows from (1) with the total field A_μ: $\big[i\gamma^\mu(\partial_\mu - ieA_\mu) - m_e\big]\Psi = 0$. Inserting the expansion for Ψ and multiplying from the left by $\Psi_m^\dagger(\mathbf{r})\gamma^0$ (to convert to Hamiltonian form) we obtain, after spatial integration,

$$\sum_n \Big[\big(i\partial_t - \omega_n\big)\delta_{mn} - \mathcal{H}^{\text{int}}_{mn}(t)\Big]c_n(t) = 0, \tag{27}$$

where the interaction matrix element $\mathcal{H}^{\text{int}}_{mn}(t)$ contains all couplings to A^{rad} and A^{ZP}.

We separate the deterministic and stochastic contributions by writing

$$\mathcal{H}^{\text{int}}_{mn}(t) = \mathcal{H}^{\text{rad}}_{mn}(t) + \mathcal{H}^{\text{ZP}}_{mn}(t).$$

Radiation reaction (deterministic part)

The self-field A^{rad}_μ satisfies the Maxwell equations with the electron current as source. For a weakly damped system, the dominant contribution comes from the on-shell part of the self-field that oscillates at the same frequency as the mode itself. Standard classical radiation theory [14] gives, in the non-relativistic approximation, $\mathcal{H}^{\text{rad}}_{mn}(t) \approx -i\gamma_n\,\delta_{mn}$, where the damping constant for mode n is

$$\gamma_n = \frac{e^2}{6\pi m_e}\sum_k |\mathbf{d}_{nk}|^2\,\omega_{nk}^3, \tag{28}$$

with $\mathbf{d}_{nk} = -e\int d^3x\,\Psi_n^\dagger\mathbf{r}\Psi_k$ and $\omega_{nk} = \omega_n - \omega_k$ (positive for emission, negative for absorption). For a stable bound state, γ_n is the total spontaneous emission rate, matching (26).

Zero-point noise (stochastic part)

The zero-point potential A^{ZP}_μ is a Gaussian random field with zero mean and covariance (19). The interaction Hamiltonian is linear in A^{ZP}_μ; projecting it onto the Dirac basis yields the complex matrix elements

$$\mathcal{H}^{\text{ZP}}_{mn}(t) = -ie\int d^3x\,\bar{\Psi}_m(\mathbf{r})\gamma^\mu\Psi_n(\mathbf{r})\,A^{\text{ZP}}_\mu(\mathbf{r},t). \tag{29}$$

Because A^{ZP}_μ contains all frequencies, these off-diagonal terms couple every pair of modes to the ZP field. However, in the rotating-wave approximation (RWA), we retain only those contributions that oscillate slowly compared to the optical frequencies. For a weakly damped system, the occupation amplitudes $c_n(t)$ vary on timescales $\sim 1/\gamma_n \gg 1/\omega_n$. The off-diagonal elements $\mathcal{H}^{\text{ZP}}_{mn}$ with $m \neq n$ oscillate at frequency $|\omega_m - \omega_n|$, which for hydrogenic transitions is of order $\sim m_e\alpha^2$, still much larger than $\gamma_n \sim m_e\alpha^5$. We may therefore perform a time-averaging (secular approximation) and drop all rapidly oscillating terms. Only the diagonal part $\mathcal{H}^{\text{ZP}}_{mm}$ survives, giving the noise term

$$f_m(t) \equiv \mathcal{H}^{\text{ZP}}_{mm}(t) = -ie\int d^3x\,\bar{\Psi}_m(\mathbf{r})\gamma^\mu\Psi_m(\mathbf{r})\,A^{\text{ZP}}_\mu(\mathbf{r},t). \tag{30}$$

The off-diagonal contributions are not strictly zero, but they average out on the damping timescale and generate only small second-order energy shifts; they do not contribute to the leading-order stochastic dynamics of the populations. [9]

Assembling the deterministic and stochastic parts and applying the RWA, the projected equations (27) reduce to a set of independent Langevin equations for the complex amplitudes:

$$\dot{c}_m(t) = -\gamma_m\,c_m(t) + f_m(t), \tag{31}$$

where $f_m(t)$ is Gaussian white noise with zero mean.

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Diffusion matrix

The autocorrelation of the noise follows from (19) and the dipole matrix elements. In the Coulomb gauge and the long-wavelength approximation ($ka_0 \ll 1$), the relevant component is the electric dipole coupling. For the diagonal noise terms we obtain

$$\langle f_m(t) f_m^*(t') \rangle = 2D_m \, \delta(t - t'), \tag{32}$$

with

$$D_m = \frac{e^2}{6\pi} \sum_k |\mathbf{d}_{mk}|^2 \, |\omega_{mk}|^3. \tag{33}$$

The sum over k runs over all stationary states. Because the integrand is symmetric, we can separate the sum into "downward" contributions ($\omega_k < \omega_m$, corresponding to decay) and "upward" contributions ($\omega_k > \omega_m$, corresponding to absorption). The downward part gives the same combination as γ_m; the upward part, which originates from the zero-point radiation, has an identical magnitude because the ZP spectrum is symmetric. Thus the noise satisfies the classical fluctuation-dissipation relation

$$D_m = \gamma_m. \tag{34}$$

The cross-correlation between different modes, $\langle f_m(t) f_n^*(t') \rangle$ for $m \neq n$, is suppressed by the RWA and is negligible in the long-time limit that concerns us here. (It can be shown that its contribution to the Fokker-Planck equation for the populations averages to zero under the same secular approximation.)

A.2 Fokker-Planck equation and uniform weight distribution

We now specialise to a superposition of two orthogonal states $|1\rangle$ and $|2\rangle$ with $\omega_2 > \omega_1$. The Langevin equations (31) for c_1, c_2 are

$$\dot{c}_1 = -\gamma_1 c_1 + f_1(t), \qquad \dot{c}_2 = -\gamma_2 c_2 + f_2(t).$$

It is convenient to write $c_m = \sqrt{I_m}\, e^{i\phi_m}$ (amplitude-phase decomposition). Using Itô calculus, the stochastic differential equations for the intensities $I_m = |c_m|^2$ read

$$dI_m = (-2\gamma_m I_m + 2D_m)\, dt + \sqrt{4D_m I_m}\, dW_m,$$

where W_m are independent Wiener processes. With the fluctuation-dissipation relation $D_m = \gamma_m$, the drift term vanishes for the stationary distribution of I_m. The total energy $E \propto \omega_1 I_1 + \omega_2 I_2$ is approximately conserved over the damping timescale (the energy lost to radiation is resupplied by the ZP). Therefore, to leading order, the sum $I_1 + I_2$ fluctuates weakly around a constant; we impose the normalisation $I_1 + I_2 = 1$ (absorbing the constant into the definition of the states).

We now define the weight $w = I_2/(I_1 + I_2) = I_2$. From the Itô equations for I_1, I_2 one derives the stochastic differential equation for w:

$$dw = (\gamma_1 - \gamma_2)\, w(1 - w)\, dt + \sqrt{4\gamma_1\gamma_2\, w(1 - w)}\, dW,$$

where we have used the fact that the two noise sources are independent and $D_m = \gamma_m$. The corresponding Fokker-Planck equation for the probability density $P(w, t)$ is

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial w}\left[D_w(w)\frac{\partial P}{\partial w}\right] - \frac{\partial}{\partial w}\left[v_w(w)P\right], \tag{35}$$

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with the diffusion coefficient

$$D_w(w) = 2\gamma_1\gamma_2\, w(1-w),$$

and the drift term $v_w(w) = (\gamma_1 - \gamma_2)w(1-w)$. For hydrogenic levels γ_1 and γ_2 are both proportional to $\alpha^5 m_e$ and are comparable. However, in the steady state the drift is balanced by the diffusion, and the stationary solution of (35) is

$$P_{\rm st}(w) \propto \exp\left(\int \frac{v_w(w')}{D_w(w')}\, dw' \right) = \text{constant}, \tag{36}$$

because $v_w/D_w = (\gamma_1 - \gamma_2)/(2\gamma_1\gamma_2)$, independent of w. Hence the stationary distribution of w is uniform on $[0,1]$. After a sufficiently long time under the influence of the zero-point field, all possible energy partitions between the two modes are equally probable.

A.3 Detection as nonlinear amplification

A macroscopic detector tuned to the transition frequency $\Delta\omega = \omega_2 - \omega_1$ couples to the oscillating dipole (22) and absorbs energy at a rate proportional to $I_1 I_2 = w(1-w)$. Crucially, any real detector possesses a nonlinear threshold: below a certain absorbed power it remains in a metastable "ready" state; above the threshold it rapidly transitions to a macroscopically distinct pointer state. This nonlinearity breaks the symmetry between the two modes. Because the zero-point noise continuously drives the instantaneous weight w across the interval $[0,1]$, the first passage of the absorbed power above the threshold selects the mode that is momentarily dominant. The probability that the detector registers outcome "2" is therefore the fraction of time that w is above the effective threshold, which, for a uniform weight distribution and a symmetric coupling, equals $\langle w \rangle = 1/2$ when $|c_1| = |c_2|$. For general initial amplitudes, the uniform distribution of the weight implies that the detection probability is exactly $P(2) = |c_2|^2$.

The argument generalises immediately to many-state superpositions: the phase space of the coefficients supports a uniform stationary measure on the sphere $\sum_n |c_n|^2 = 1$, and the detector's energy absorption picks out the component with the largest instantaneous weight, yielding the full Born rule $P(n) = |c_n|^2$.

A.4 Summary

The zero-point radiation provides a dephasing mechanism that erases the deterministic interference pattern, while the detector's nonlinearity converts the continuous classical superposition into a discrete macroscopic outcome. The probabilities are not fundamental but emerge from the ergodic properties of the stochastic dynamics, exactly as in the SED programme for Schrödinger particles. No wave-function collapse or quantum postulate is required; the Born rule is a consequence of the interplay between the classical field equations and the ever-present zero-point background.

References

[1] H. A. Lorentz, *The Theory of Electrons*, 2nd ed. (Teubner, Leipzig, 1916).

[2] M. Abraham, "Prinzipien der Dynamik des Elektrons," *Ann. Phys.* **315**, 105 (1903).

[3] E. Schrödinger, "Quantisation as a Problem of Proper Values (Parts I–IV)," *Ann. Phys.* 79, 361 (1926); 79, 489 (1926); 80, 437 (1926); 81, 109 (1926).

[4] E. Schrödinger, *Collected Papers on Wave Mechanics* (Blackie & Son, London, 1928).

[5] L. de Broglie, "Recherches sur la théorie des quanta," *Ann. Physique* (Paris) **3**, 22 (1925).

[6] L. de Broglie, *Non-linear Wave Mechanics: A Causal Interpretation* (Elsevier, Amsterdam, 1960).

[7] D. Bohm, "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables, I and II," *Phys. Rev.* **85**, 166, 180 (1952).

[8] T. H. Boyer, "Random Electrodynamics: The Theory of Classical Electrodynamics with Classical Electromagnetic Zero-Point Radiation," *Phys. Rev. D* **11**, 790 (1975).

[9] L. de la Peña and A. M. Cetto, *The Quantum Dice: An Introduction to Stochastic Electrodynamics* (Kluwer, Dordrecht, 1996).

[10] H. E. Puthoff, "Ground State of Hydrogen as a Zero-Point-Fluctuation-Determined State," *Phys. Rev. D* **35**, 3266 (1987).

[11] A. O. Barut, "Electrodynamics and Bound States," in *Quantum Electrodynamics and Quantum Optics*, edited by A. O. Barut (Plenum, New York, 1984), pp. 13–34.

[12] A. O. Barut and A. J. Bracken, "Zitterbewegung and the Internal Geometry of the Electron," *Phys. Rev. D* **23**, 2454 (1981).

[13] A. O. Barut and J. P. Dowling, "Quantum Electrodynamics Based on Self-energy: The Lamb Shift and Spontaneous Emission Without Vacuum Fluctuations," *Phys. Rev. A* **36**, 649 (1987).

[14] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).

[15] P. Pearle, "Classical Electron Models," in *Electromagnetism: Paths to Research* (Wiley, New York, 1982).

[16] F. Rohrlich, *Classical Charged Particles*, 3rd ed. (World Scientific, Singapore, 2007).

[17] P. A. M. Dirac, "Classical Theory of Radiating Electrons," *Proc. R. Soc. Lond. A* **167**, 148 (1938).

[18] J. A. Wheeler and R. P. Feynman, "Interaction with the Absorber as the Mechanism of Radiation," *Rev. Mod. Phys.* **17**, 157 (1945).

[19] S. Coleman, "Q-Balls," *Nucl. Phys. B* **262**, 263 (1985).

[20] R. Friedberg, T. D. Lee, and A. Sirlin, "Class of Scalar-Field Soliton Solutions in Three Space Dimensions," *Phys. Rev. D* **13**, 2739 (1976).

[21] A. Kusenko, "Solitons in the Supersymmetric Extensions of the Standard Model," *Phys. Lett. B* **405**, 108 (1997).

[22] I. Bialynicki-Birula and J. Mycielski, "Uncertainty Relations for Information Entropy in Wave Mechanics," *Commun. Math. Phys.* **44**, 129 (1975).

[23] T. Takabayasi, "On the Formulation of Quantum Mechanics associated with Classical Pictures," *Prog. Theor. Phys.* **8**, 143 (1952).

[24] D. Bohm and J. P. Vigier, "Causal Interpretation of Quantum Theory in Terms of a Fluid with Irregular Fluctuations," *Phys. Rev.* **96**, 208 (1954).

[25] E. Nelson, "Derivation of the Schrödinger Equation from Newtonian Mechanics," *Phys. Rev.* **150**, 1079 (1966).

[26] A. Einstein, "Quanten-Mechanik und Wirklichkeit," *Dialectica* **2**, 320 (1948).

[27] J. S. Bell, "On the Einstein–Podolsky–Rosen Paradox," *Physics* **1**, 195 (1964).

[28] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum, New York, 1977).

[29] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[30] M. Yaremenko, *London J. Phys.* **3**, 18057 (2026).

[31] P. A. M. Dirac, *Proc. Roy. Soc. Lond. A* **117**, 610 (1928).

[32] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, 1930).

[33] W. Pauli, *Z. Phys.* **43**, 601 (1927).

[34] L. L. Foldy and S. A. Wouthuysen, *Phys. Rev.* **78**, 29 (1950).

[35] L. de la Peña and A. M. Cetto, *Found. Phys.* **25**, 907 (1995).

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