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Evolution of Stellar Rotation in Open Clusters from NGC2547 (35Myr) to NGC6811 (950 Myr)

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Abstract

Understanding stellar rotation and the role of magnetic braking remains a major outstanding problem in astrophysics. In this paper, stellar rotation of the young open cluster Blanco 1 is investigated and compared with other clusters with ages ranging from ∼35 Myr to ∼950 Myr and star masses in the range $0.2 < M_\odot < 1.4$. It is proposed that rotation rates of stars in young open clusters are determined by the early angular momentum acquired during formation and not by magnetic braking effects, which operate on longer time scales. On the convective C Sequence, for lighter stars, the early angular velocity is related directly to the moment of inertia. However, for the interface or I Sequence, differential rotation exists and observed rotation rates are essentially the rotation rates of the outer convection zone. It is estimated that the convection zone can rotate at over 10× the rate of the inner radiative zone for heavier stars. This differential rotation process can drive saturated interface dynamos that lead to large rotational mass dependences which can be described by a simple energy transfer model. Comparing with clusters of different ages magnetic braking outlines the evolution of older clusters from young clusters. For old clusters such as Praesepe the majority of stars have experienced spin down, that can be described by a simple model based on an additional momentum loss proportional to the angular momentum of the early cluster, which for the I Sequence is consistent with the well known Skumanich relation.

1 Introduction

The evolution of stellar rotation has been studied for several decades but remains an unresolved issue in astrophysics. Stars with different masses have different measured rotational rates depending on age and the origins of this behaviour are unclear. Observational evidence suggests low mass stars are born with rotation rates in the range ∼ 1–10 days and subsequent star disc coupling times occur over ∼ 10 Myr [1] [2]. On the zero-age main sequence (ZAMS) braking by magnetised winds reduces the stellar rotational periods [3] [4] [5] and results in spin down to lower angular velocities. However, the role of initial conditions and the further evolution of older clusters remains uncertain particularly since the stellar rotation is observed to be strongly mass dependent [6] [7].

It has been proposed that a knowledge of stellar rotation and spin down will aid in gyrochronology [8] [9] [10]. Models of spin down processes will aid in determining the age of stars particularly when other observable stellar properties change slowly with time. Also, recently the role of exoplanet host stars and their characterisation [11] has become important for establishing planetary habitable zones [12]. The role of UV/X-ray stellar radiation and flares on exoplanets and potential biomolecule destruction [13] [14] and atmospheric erosion [15] are related to the stellar magnetic fields associated with stellar rotation and dynamo processes.

In current theories of stellar magnetic braking, the magnetic field couples to the magnetised stellar wind to reduce stellar angular momentum over long time periods [16] [17] [18] [19] [20]. The rate of angular momentum loss is principally related to the angular velocity, dynamo type and magnetic field geometry. The characteristic spin down period or Skumanich timescale of the Sun is estimated to be around 1 Gyr

[21]. Thus, for young stars ∼100 Myr of similar mass the impact of magnetic braking is expected to be low and other physical processes must determine the rotational behaviour. The long term magnetic braking of stars according to the Skumanich relation $P \propto t^{1/2}$ [26] relating periodicity P to time t is observationally well established. However, the initial rotation conditions and subsequent evolution towards the Skumanich relation can now be investigated using precision rotation data from star clusters of varying ages. Particularly interesting is the cross sectional rotation behaviour of clusters where periodicity varies with spectral colour or mass which I explore in this paper.

1.1 The Cluster Data

Blanco 1 was first discovered by Blanco (1949) and is a young open cluster ∼115 Myr old situated in the local spiral arm ∼240 pc in the direction towards and below the Galactic centre with an unusual high Galactic latitude. It is comprised of 489 Gaia DR2-confirmed stars, ranging from B-to-M spectral types. Blanco 1 is a smaller version of the Pleiades (∼110-125 Myr, 1326 Gaia DR2 members) with a lower stellar density [22].

The Next Generation Transit Survey (NGTS) data of the Blanco 1 cluster [22] is a unique dataset using around 200 days monitoring of 127 stars of spectral types F5-M3 with $1.2 - 0.3 M_\odot$ and rotation periods from ∼1-10 days. In Gillen et al. [22] the light curve data has been fitted using a variety of numerical techniques including Gaussian Process, box transformations and autocorrelation functions to obtain rotation data to high precision. Stellar masses were determined by spectral type and updated information from Pecault & Mamajek [23]. Gillen et al. [22] provided no theoretical interpretation of the rotation versus mass data but suggested the role of possible magnetic braking in the observed difference in rotation rates with mass.

In stellar rotational studies, Barnes [9] [24] divided the rotation period versus B-V colour or mass into the I Sequence and the C Sequence. The I Sequence or interface sequence is for stars possessing a proposed interface dynamo type. The envisaged dynamo is formed at the interface of the differentially rotating convective and radiative zones. The I Sequence terminates near the stellar mass where full convection sets in and the radiative/convective boundary in the star vanishes. The C Sequence describes a convective dynamo process, possibly weaker than the interface dynamo of heavier stars.

Figure 1 shows data from NGTS for Blanco 1 [22]. The higher mass stars on the I Sequence exhibit decreasing rotational speed with decreasing mass along a well defined curve until around $0.8 M_\odot$. For masses below $0.8 M_\odot$ on the C Sequence the rotational speed increases and the speed as a function of mass displays a large scatter or variance.

The Blanco 1 results are compared with fitted Gaia data from Gedoy-Rivera [25] and the Monitor Project. The Gaia data came from the DR2 data release and is used to help determine membership of 7 clusters, for which rotational data was previously measured using ground and space based techniques, ranging in age from ∼35 Myr (NGC 2547) to ∼950Myr (NGC 6811). A cluster fitting approach similar to membership probability studies was used to determine cluster members. In the work used here I only use "probable members" that have a theoretical membership probability of greater than 90 %.

The combination of the NGTS data and the Gaia data allows insights into the evolution of cluster rotation with age. In this paper I develop simple theoretical power laws to describe the observed mass dependence of the periodicity of rotation. The rotation data is plotted as logarithms of period P versus normalised mass M such that a rotation law of the form $P = AM^x$ becomes a linear form $\log P = x \log M + C$. The variable C becomes an arbitrary fitting constant and the primary focus is elucidate the physical processes determining x common to the observed clusters. A simple theoretical description of the rotational behaviour and the variation with mass of the C and I Sequences is outlined. The C Sequence rotation is found on average to be dependent on the initial angular momentum conditions and the total moment of inertia. The I Sequence rotation is based on the observable differential rotation of the outer convection zone and is related to the moment of inertia of the convective zone, which becomes thinner for heavier stars. An equilibrium condition based on energy flux, when the interface dynamo is fully saturated, is developed for the I Sequence and provides good agreement with experimental data. It is discovered while that early clusters have rotation behaviour based on their initial angular momentum, for the older clusters the periodicity is modified by angular braking and deceleration. For these older clusters there is good agreement with a simple spin down model and the Skumanich relation. However, for heavier stars fast differential rotation is continued to be observed.

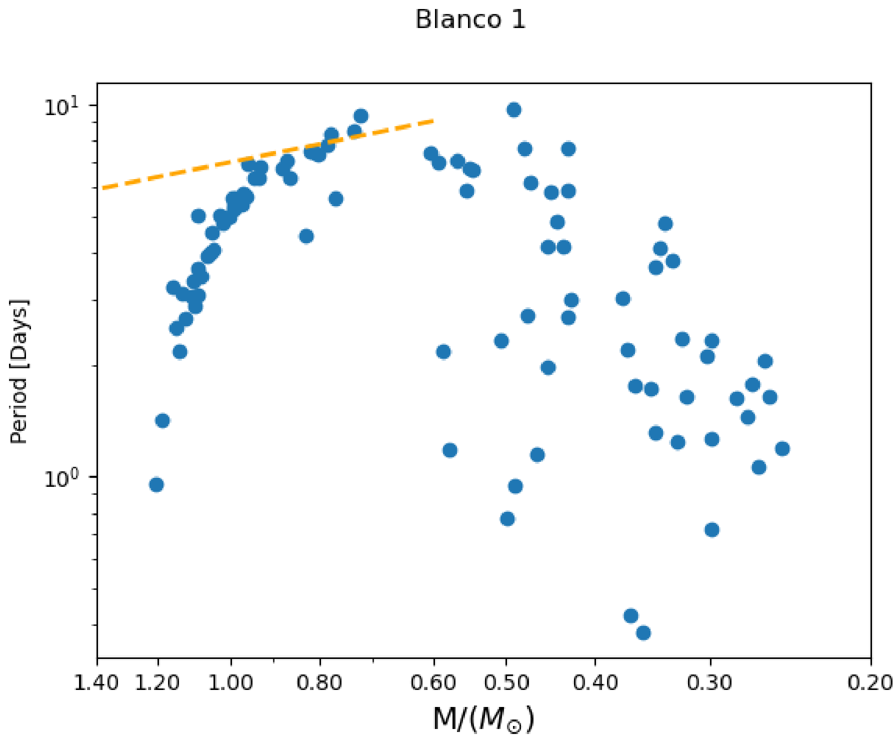

Figure 1: Rotational Period of Blanco 1 from NGTS data vs. Mass. The yellow dotted line shows the periodicity versus $M^{-1.2}$ power law from Eq.(7) for the I Sequence.

1.2 Review of Magnetic Braking Models

The conventional process of loss of angular momentum is either coupling of the magnetic field to an outer plasma or stellar wind or ionised disc remnants that corotate losing mass and hence angular momentum at larger distances. The coupling of the surface of the star to a magnetised wind can be typically described as a rate of change in the angular momentum [21]. The simplest model specification of Weber [20] assumes a spherical loss of angular momentum J from a mass loss rate $\dot{M}$ at an Alfvén radius r_A

$$\frac{dJ}{dt} = -\frac{2}{3}\Omega\dot{M}{r_A}^2 = -\frac{J}{\tau_J}, \tag{1}$$

where $\Omega = 2\pi/P$ is the stellar angular velocity and τ_J the spin down time. More complex braking models include specifications of the form of the magnetic field and the dynamo process. Typically the field B_r at a distance r from the star of radius R is given as

$$\frac{B}{B_r} = (\frac{r}{R})^n, \tag{2}$$

where n varies for radial or dipole fields and B is the surface field. Most disc models assume an power law relation between the stellar magnetic field and the angular velocity [21]

$$B \propto \Omega^\alpha, \tag{3}$$

where α is conventionally taken as unity. However, for a fast rotating star with dynamo in the magnetostrophic regime the angular velocity is theoretically closer to being proportional to B^2 according to recent studies [27].

The equations of Kawaler [18] are often used as the basis for the rate of angular momentum loss due to magnetic fields

$$\frac{dJ}{dt} = -K\Omega^{1+4N/3}(\frac{R}{R_\odot})^{2-N}(\frac{M}{M_\odot})^{-N/3}(\frac{\dot{M}}{10^{-14}})^{1-2N/3}, \tag{4}$$

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where $N = \alpha n$. To recover the original Skumanich relation [26] requires setting $N = 1.5$ and retaining only the leading Ω^3 term so that

$$\frac{dJ}{dt} \propto -\Omega^3. \tag{5}$$

The requirement of $N = 1.5$ is midway between a linear $n = 2$ and dipole $n = 3/7$ field with a linear dynamo relation $\alpha = 1$. Integrating and assuming constant moment of inertia I gives the Skumanich relation

$$\Omega \propto \frac{I^{1/2}}{t^{1/2}}. \tag{6}$$

If the stellar radius $R \propto M^{0.7}$ [24] the moment of inertia $I \propto M^{2.4}$ then the mass variation is

$$\Omega \propto M^{1.2}. \tag{7}$$

Notably, this M dependence is too weak to fit the experimental data for Blanco 1 and is shown as the yellow dotted line in Figure 1.

Above a certain angular threshold velocity Ω_{thres} it is often proposed that the loss rate must saturate [21] [28]. Modifications to the Kawaler equation lead to the form

$$\frac{dJ}{dt} = -K\Omega{\Omega_{thres}}^{4N/3}(\frac{R}{R_\odot})^{2-N}(\frac{M}{M_\odot})^{-N/3}(\frac{\dot{M}}{10^{-14}})^{1-2N/3}. \tag{8}$$

Again considering only the Ω variation then

$$\frac{dJ}{dt} \propto -\Omega, \tag{9}$$

which is linear in Ω. This form can be integrated to provide an exponential form which as pointed out by Barnes [24] does not well fit observational data since it predicts decreasing rotation rates with decreasing mass. On the C Sequence measured rotation rates generally increase with decreasing mass as in Figure 1 for $M < 0.6$.

Studies indicate that rotational saturation effects may be evident with typically $\Omega_{thres} > 10\Omega_\odot$, or $\Omega_{thres} > 5\Omega_\odot$ for differential rotation [21]. In addition, coronal X-ray observations [29] [30] [32] demonstrate a clear saturation effect with Rossby number Ro and angular velocity. The Rossby number is $Ro = P/\tau_{conv}$ where τ_{conv} is the convective turnover time. This saturation may correlate with the proposed saturation effects in the angular momentum process. The empirical measurements of X-ray L_X and bolometric luminosity L show a relation with the Rossby number Ro

$$\frac{L_X}{L} \propto Ro^{-\gamma} \propto \Omega^\gamma. \tag{10}$$

Typically when coronal saturation exists for faster rotating stars then $\gamma = 0$ and

$$L_X \propto L. \tag{11}$$

The magnetic braking theory outlined above does not fit the cluster data for Blanco 1 in Figure 1. This suggests that although magnetised winds might decrease angular velocity over longer periods of time for older clusters they do not determine the initial rotational curve observed in young clusters. Also, the basic theory is perhaps more relevant to stars rotating as solid bodies and not exhibiting differential rotation.

2 Rotation in Young Clusters

Before studying the evolution of rotation in clusters of different ages it is informative to investigate the processes that establish the initial rotation rates in the younger clusters in the dataset. Blanco 1 and Pleiades provide two young clusters of similar age for which high quality rotational data is available. Here it is considered, whether differential rotation can be measured and power law models developed for the C Sequence and I Sequence to describe the observed data.

2.1 Differential Rotation for Blanco 1 and Pleiades

For the young stars in Blanco 1 it is likely that the early rotation rates are determined by the initial angular momentum acquired during star formation rather than slower magnetic braking effects. Following Barnes [24] for Figure 1 the region with approximately $M < 0.8M_{\odot}$ can be considered as corresponding to the C Sequence and higher masses $M > 0.8M_{\odot}$ the I Sequence. On the I Sequence the stars have thinner convective zones (CZ) with increasing mass whereas on the C Sequence the stars approach fully convective behaviour.

The high angular velocities of the I Sequence are proposed to be due to differential rotation [9], [24]. Since the thickness of the convective zone (CZ) falls with increasing mass the angular velocity of the CZ shell can grow with larger stellar mass on the I Sequence. Figure 2 shows the moment of inertia using the Yale stellar code (YREC) from Barnes [9] for the core, radiative zone (RZ), CZ and total moment of inertia for a 500 Myr model of solar composition. For the precision of the following calculations, employing these theoretical moment of inertia, any stellar evolution is expected to have little impact on the results and conclusions.

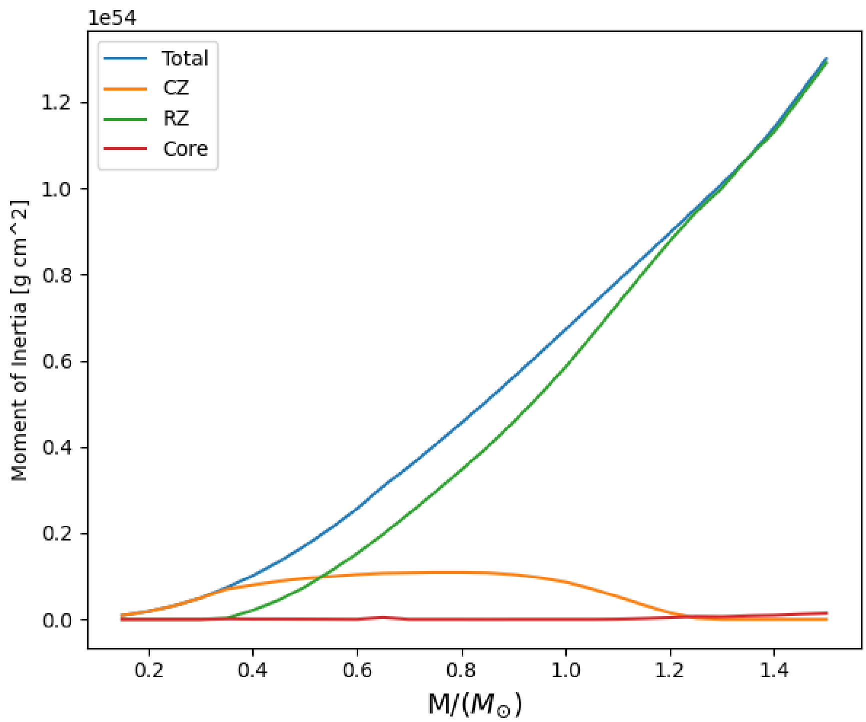

Figure 2: Theoretical Moment of Inertia vs. Mass.

The general form of the moment of inertia curves can provide some illumination as to the distribution of angular momentum for early stars. In Figure 3 is plotted for Blanco 1 and Pleiades the angular velocity forms $\frac{A}{I_{CZ}}$ (yellow) and $\frac{B}{I_{TOT}}$ (red) where I_{CZ} is moment of inertia of the CZ and I_{TOT} the total moment of inertia with A and B are arbitrary fitted constants.

Since angular momentum is usually expressed as a product of a moment of inertia and angular velocity, these forms demonstrate that the change in the moment of inertia of the CZ (yellow line) can accommodate the observed behaviour in the I Sequence without recourse to other physical braking processes, provided the differential rotation exists and the CZ is weakly coupled to the RZ. The crossover in rotational behaviour from the I Sequence to the C Sequence would then be primarily driven by the moment of inertia of the entire star (red line) when the CZ becomes tightly coupled to the RZ. Helioseismic observation of the Sun show it behaves to zeroth order as a solid body for rotation [21]. In the Blanco 1 cluster it might be expected that similar solid body effects dominate in fully convective stars. Thus at values around $0.8M_{\odot}$ the star becomes

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more convective and this is demonstrated in the rotational behaviour.

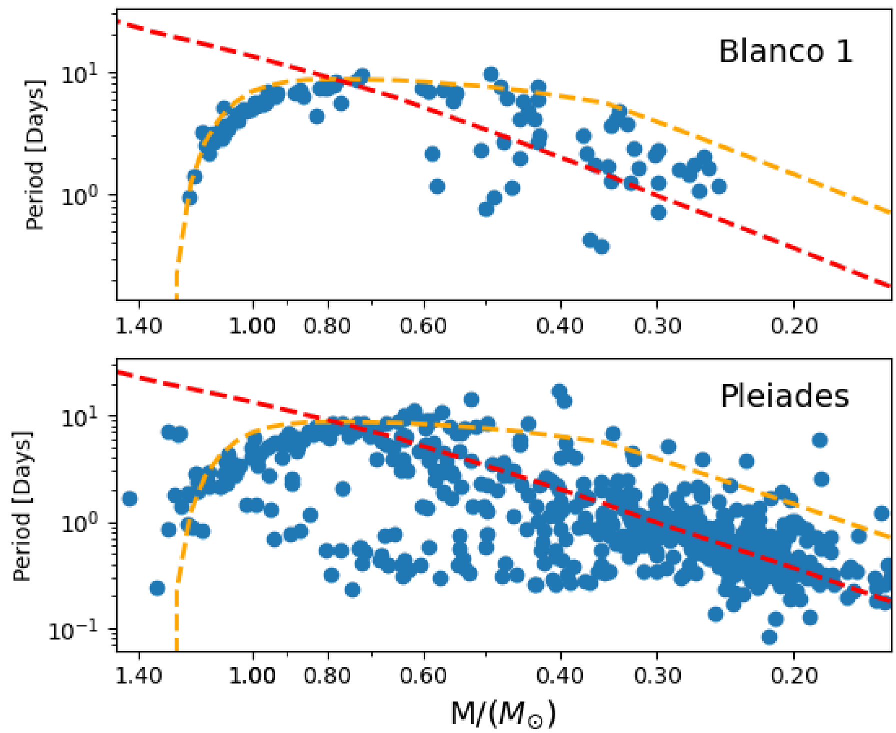

Figure 3: Rotational Period of form $\Omega = \frac{A}{I_{CZ}}$ (yellow line) and $\frac{B}{I_{TOT}}$ (red line) vs. Mass for Blanco 1 and Pleiades.

2.2 Estimating the Differential Rotation of Blanco 1 and Pleiades

Theoretically, neglecting the relatively insignificant core, the total angular momentum J_{TOT} can be written as a sum of the RZ and the differentially rotating CZ as

$$J_{TOT} = I_{CZ}\Omega_{CZ} + I_{RZ}\Omega_{RZ} = I_{TOT}\Omega_{TOT} \tag{12}$$

where Ω_{CZ}, Ω_{RZ} and Ω_{TOT} are the angular velocities for the CZ, RZ and if the star behaved as a solid body. For solid body rotation the angular velocities are then matched

$$\Omega_{CZ} = \Omega_{RZ} = \Omega_{TOT}. \tag{13}$$

The cluster of Blanco 1 is a young cluster and if little recoupling or magnetic braking can be assumed so the elapsed time t is less than any stellar spin down time τ_J and any CZ/RZ recoupling time τ_c. Considering that the total early angular momentum for mass M can be approximated as that of the average angular momentum when the star mass becomes a solid body M_{SB} then

$$J_{TOT}(M) \approx J_{TOT}(M_{SB}) = J_{TOT}, \tag{14}$$

where $M_{SB} \approx 0.8M_\odot$ from Figure 3. This approximation requires that any braking factor for the young cluster is independent of the mass but could be varying in time so any mass and time variation will then cancel in both sides of the above equation. The total angular momentum can be estimated as the observed angular momentum $J_{TOT}(M_{SB})$ using the theoretical I_{TOT} from Figure 2 and the observed average rotation rate at M_{SB}. This provides an estimate for the total angular momentum as $J_{TOT} \approx 5.1 \times 10^{48} gcm^2s^{-1}$.

The RZ rotation rate for other masses in the I Sequence can then be estimated using the observed values of Ω_{CZ} and theoretical moment of inertia as

$$\Omega_{RZ} = \frac{J_{TOT} - I_{CZ}\Omega_{CZ}}{I_{RZ}}. \tag{15}$$

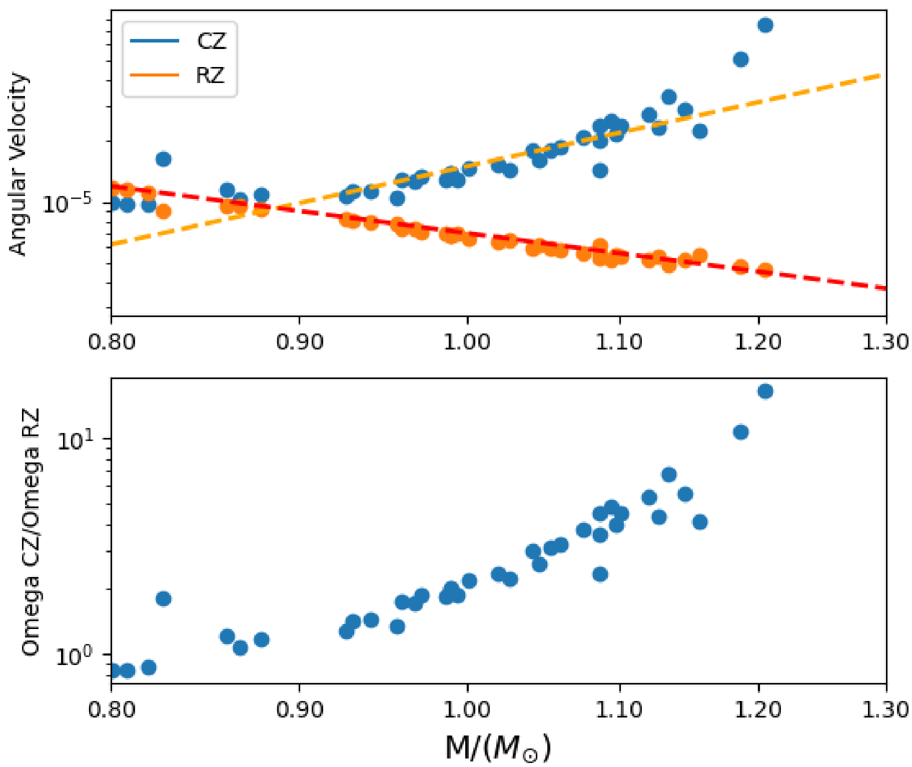

Figure 4: (Upper) Observed angular velocity of convective zone and estimated angular velocity of radiative zone vs. Mass. The blue circles are the observed angular velocity of the convective zone and the orange circles are the estimated values for the radiative zone. The yellow dotted line is a fit of $\Omega_{CZ} \propto M^4$ and the orange dotted line $\Omega_{RZ} \propto M^{-2.4}$. (Lower) Ratio of convective zone and radiative zone angular velocities vs. Mass.

By assuming that at around $M \approx 0.8M_\odot$ the star becomes more convective and behaves as a solid body and that on average the initial angular momentum across all stars in the cluster is equivalent we can separate the CZ and RZ rotations. From the Blanco 1 data the observed values of Ω_{CZ} and the estimated values of Ω_{RZ} are shown in Figure 4 as a function of mass. In the lower Figure 4 the CZ rotation rates vary between $1\times$ to $10\times$ the RZ rotation rates. In upper Figure 4 the yellow line shows the approximate Ω_{CZ} power law of

$$\Omega_{CZ} \propto M^4, \tag{16}$$

and the orange line Ω_{RZ} shows a relationship of

$$\Omega_{RZ} \propto M^{-2.4} \propto I^{-1}. \tag{17}$$

A RZ power law of the form of Eq. (17) might be expected if the RZ, which forms the majority of the moment of inertia, is treated as a solid body.

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2.3 Discussion of the Estimated Differential Rotation of Blanco 1 and Pleiades

In the above model described by Eq. (15), the estimated angular velocity of the CZ is higher than that of the internal RZ for early clusters on the I Sequence. It is conventionally assumed [18] that, due to the lower moment of inertia, the CZ is more susceptible to magnetic braking effects and that the CZ perhaps rotates more slowly than the RZ [24]. Although, this may be the case for later clusters the above results suggest it is not true for the early clusters.

If for the early cluster I Sequence, the observed CZ angular velocity was less than for the RZ then very large angular momentum changes would be required for very similar masses. For the Blanco 1 limiting case if $\Omega_{CZ} = \Omega_{RZ}$ then between $1.0M_{\odot}$ and $1.3M_{\odot}$ there would need to be an earlier angular momentum change of over $10\times$. This would arguably require a more complex external physical process than a simple early internal rearrangement of angular momentum leading to $\Omega_{CZ} > \Omega_{RZ}$. The simplest explanation for Blanco 1 and Pleiades is that there was an early internal redistribution of angular momentum. This process could have been during the formation of the CZ and RZ, during the earlier stellar radius changes or driven by the existence of an interface dynamo.

In the calculation of Eq. (15) it is assumed that the stars behave as solid bodies at $0.8M_{\odot}$. The choice of 0.8 is subjective but supported by the fact that it is widely viewed [9] [21] that by $0.8M_{\odot}$ convective behaviour is established and it is also approximately the observed change in regime between the I and C Sequences in Figure 3. Notably, the fitted power laws in Figure 4 are not sensitive to the exact choice of $0.8M_{\odot}$.

A second important assumption in the calculation is that, for the early clusters, the mean angular momentum is weakly dependent on mass and that as an approximation the same total angular momentum can be used in Eq. (15) as was calculated for $0.8M_{\odot}$. Retrospectively, this assumption seems to be a good one since at 100Myr the observed mean angular momentum for the C Sequences (see Section 3.6) and fitted for the estimated RZ angular velocity as Eq. (17) follow that roughly expected for a solid body.

The variation of total angular momentum as a function of mass for early clusters is an open question. Studies on disc locking and early braking commonly use one of two assumptions as their starting conditions [31]. Either setting the starting angular momentum $\propto M^{0.985}$ as estimated by Kawaler [33] for early stars (across all masses) or setting the angular velocity to a specific value of say 5 or 8 days, which results in the angular momentum strongly varying with mass. Both of these approaches are perhaps unsatisfactory, particularly in the presence of differential rotation and would not fit the observed data for Blanco 1 and Pleiades.

A basic model where a fraction of the angular momentum of a coupled proto disc/star system can provide an early cluster angular momentum that is weakly dependent on mass is outlined in Eq. (20) below. Provided the product of the angular momentum of the proto disc and the fraction that is transferred to a star are random distributions and uncorrelated to the final stellar mass, then the average total stellar angular velocity should be inversely proportional to the angular momentum. Importantly, the angular velocity of individual stars will vary, as observed on the C Sequence, but the average starting angular momentum will not vary wth mass.

The difference between the CZ and RZ angular velocities $\Delta\Omega = \Omega_{CZ} - \Omega_{RZ}$ is the differential rotation. Possibly the actual important dynamo rotation is the differential angular velocity and frequently the dynamo number [34] is written as the relative differential angular velocity $\Delta\Omega/\Omega$. At higher masses when $I_{RZ} >> I_{CZ}$ then $\Omega_{CZ} > \Omega_{RZ}$ and the rotation rate of the CZ becomes close to the differential angular velocity $\Delta\Omega \approx \Omega_{CZ}$ as the RZ rotates slowly relative to the fast CZ. Thus for higher masses the dynamo rotation rate is approximately the convective zone rotation rate and $\Delta\Omega/\Omega$ tends to unity.

2.4 The C Sequence Model

The C Sequence for the early clusters can be modelled by angular momentum conservation. Stars that have convective or fully convective behaviour lie on the C Sequence. For Blanco 1 this represents lighter stars with $M < 0.8M_{\odot}$. These lighter stars in the young cluster behave rotationally as solid bodies with the convective and radiative zones well coupled. For a spherically homogenous star with angular velocity Ω and radius R the total angular momentum J_{TOT} is

$$J_{TOT} \propto MR^2\Omega. \tag{18}$$

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Assuming $R \propto M^{0.7}$ [24] the relation to mass is then

$$J_{TOT} = kM^{2.4}\Omega, \tag{19}$$

where k is a constant. If at a later time of observation a fraction f of a proto disc angular momentum J_{D0} has been converted into the star's angular momentum

$$\Omega = \frac{fJ_{D0}}{kM^{2.4}}. \tag{20}$$

From the above equation, although the initial disc angular momentum might vary according to a probability distribution and the fraction f may vary, on average the C Sequence angular velocity is then

$$\Omega_C \propto M^{-2.4}. \tag{21}$$

2.5 The I Sequence Model

The I Sequence is a region of strong dynamo behaviour with differential rotation. Modern models of hydrodynamic dynamo's place the origin of the stellar dynamos at the interface of the radiative (RZ) and convective zones (CZ) called the tachocline [35]. For Blanco 1 the I Sequence where $M > 0.8M_\odot$, the rotation demonstrates a higher power law behaviour in the mass than suggested by Eq. (5) for pure magnetic braking determining the rotational behaviour. The smoothness of the I Sequence and the lower variance of rotational speeds relative to the C Sequence implies an energetic equilibrium is reached.

A simple dynamo model can be formed by considering how such a possible near equilibrium is created by energy fluxes and the dynamo process resulting from differential rotation. If the star is rotationally in quasi equilibrium with its environment then the local changes in angular momentum are small. The magnetic field generated by the stellar dynamo couples to and heats the stellar corona and is related to the luminosity and which can be readily related to the mass of the star. A starting point is the Elsasser number Λ which is often used to characterise dynamo behaviour [36]. For a dynamo of angular velocity Ω_D and field B_D, the number measures the ratio of magnetic forces to the Coriolis forces and is near unity for a saturated regime [27] [37].

$$\Lambda = \frac{\sigma B_D^2}{\rho \Omega_D}, \tag{22}$$

The Ω_D in the above relation is the dynamo angular velocity which can be assumed to be the observed angular velocity of the surface convective zone Ω_{CZ}. The assumption is probably reasonable in the I Sequence where $I_{CZ} << I_{RZ}$ and $\Omega_{CZ} > \Omega_{RZ}$ and the differential rotation is $\Omega_{CZ} - \Omega_{RZ} \approx \Omega_{CZ} = \Omega$. If the magnetic field of the dynamo B_D is assumed to be proportional to the surface field B then following Eq. (3)

$$\Omega \propto B^{1/\alpha}, \tag{23}$$

with $\alpha = 1/2$. The corona is heated by the energy flux which can be written as a Poynting flux

$$S_z = \frac{c}{4\pi}(\vec{E} \times \vec{B})_z \propto B^\beta. \tag{24}$$

Two major proposed coronal heating models [30] [38] are Alfvén waves with $\beta = 1$ and Parker's "nanoflares" with $\beta = 2$. Assuming the coronal luminosity L_c is proportional to the energy flux

$$L_c \propto S_z \propto B^\beta \propto \Omega^{\alpha\beta}. \tag{25}$$

which suggests $L_c \propto B^2$ for $\beta = 2$ as Fisher [32]. Empirical measurements [29] [30] [32] of X-ray and bolometric luminosity L show a relation Eq. (10) and if the X-ray luminosity is taken as the coronal luminosity $L_c = L_X$ giving

$$\Omega \propto L^{1/(\alpha\beta-\gamma)}. \tag{26}$$

Now if assuming the standard luminosity result for stars in the Blanco 1 mass range of $L \propto M^\delta$ where $\delta \approx 3.5 - 4$ [21] the angular velocity can be written

$$\Omega \propto B^{1/\alpha} \propto M^{\delta/(\alpha\beta-\gamma)}. \tag{27}$$

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Thus a saturated dynamo with $\alpha = 1/2$, nano flare coronal heating $\beta = 2$, saturated corona $\gamma = 0$ and $\delta = 4$ would have a mass dependence as

$$\Omega \propto M^4. \tag{28}$$

Alternatively, for Alfvén heating with $\beta = 1$ a higher power law would exist

$$\Omega \propto M^8. \tag{29}$$

In the above analysis assumes that (i) the dynamo is saturated so the Elsasser number Λ is around unity and (ii) the mean magnetic field in the photosphere (surface) is proportional to the dynamo magnetic field $B \propto B_D$ and (iii) coronal activity is saturated so the ratio of the X-ray luminosity to the bolometric luminosity is flat $L_X \propto L$.

The Elsasser number arises from a consideration of equipartition of the magnetic energy and the Coriolis force in the induction equation and the Navier Stokes equation and is a characteristic magnitude fairly independent of the dynamo details. Calculations by Augustson [27] on dynamo scaling relationships show that in rapidly rotating stars the dynamos reach a quasi-magnetostrophic state where the the balance between the Lorentz and Coriolis forces approaches unity and the corresponding Elsasser number is close to unity.

The magnetic fields of the dynamo consist of polodial and torodial fields most of which are internal to the star and cannot be observed. There is no theoretical consensus on the exact form of the dynamo but a leading contender is a field produced by shear at the CZ/RZ interface where magnetic A and B fields are fitted using wave equations to produce a localised hydromagnetic dynamo wave [35]. Dynamo calculations by Orvedahl [36] however show that the magnetic field at the surface is proportional to these internal fields.

The coronal heating by the photospheric magnetic fields is proportional to the coronal luminosity and magnetic flux $L_c \propto \Phi^\beta \propto B^\beta$ which is proportional to the X-ray luminosity $L_c \propto L_X$ used in coronal measurements [30]. For stars with periods less than around 10 days the ratio of the X-ray to the bolometric luminosity is observed to saturate [29] at around $L_X/L \approx 10^{-3}$. The reason for this saturation is suggested to be that shear timescales in the dynamo are less than convective turnover times.

2.6 Discussion for Blanco 1 and Pleiades

We can compare the observed periodicities for Blanco 1 and the similarly aged Pleiades with the theoretical predictions for the I and C Sequences. In Figure 5 are plotted the rotation period versus mass for the clusters.

On the C Sequence, reduction of moment of inertia due to decreasing mass will increase the angular velocity. Importantly, the amount of angular momentum transferred to the early star is random and the angular velocities of the C Sequence will exhibit high scatter or variance as observed in Figure 5. For the C Sequence the data is scattered but exhibits a rough power law based on a decreasing moment of inertia as derived in Eq. (21). The purple line shows the C Sequence power law as Eq. (21)

$$\Omega_C \propto M^{-2.4}, \tag{30}$$

For the I Sequence dynamo model is plotted the orange line

$$\Omega_I \propto M^4, \tag{31}$$

as Eq. (28) for a nanoflare model. For an Alfvén wave model at higher mass the steeper red line as Eq. (29) is shown

$$\Omega_I \propto M^8. \tag{32}$$

For the I Sequence the orange line closely fits the data suggesting a nanoflare model for coronal heating [38]. For the larger mass stars with a higher rotation speed a higher power law provides a better fit and this could indicate a transition to Alfvén wave process as the dominant coronal heating process. A flare model, however, is arguably most suitable for the majority of the young stars and is consistent with the high stellar activity in the light curves which visibly include larger flares [22].

In summary, on the I Sequence fast rotation produces a saturated magnetostrophic regime dynamo, creating a powerful magnetic field. The magnetic field heats the corona to a saturation level proportional to the luminosity. The overall luminosity is related to the mass of the star and the stellar energy source. Thus

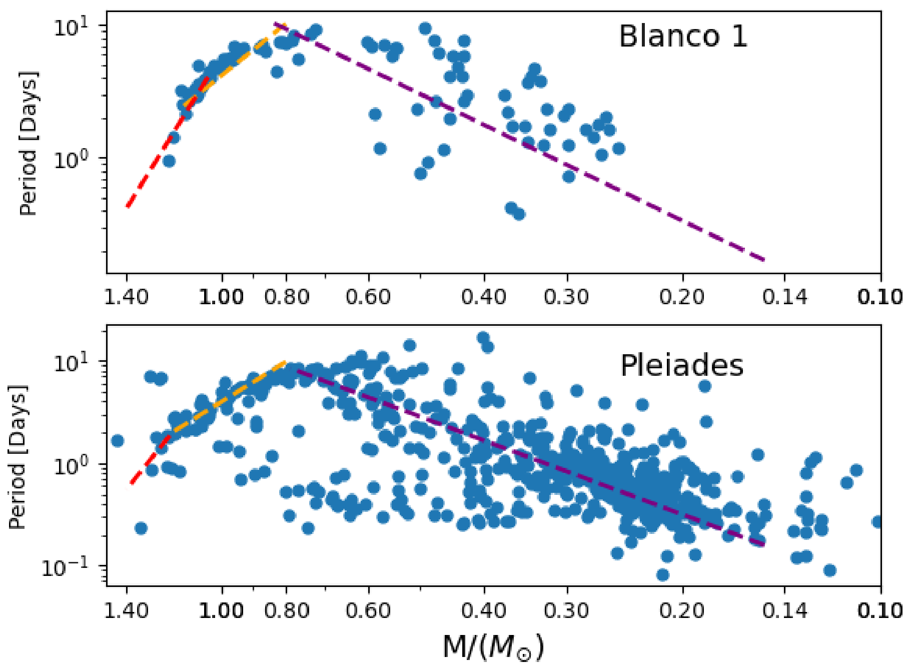

Figure 5: Rotation period vs. Mass. Blue is observed data and dotted lines refer to text.

the rotation, the saturated dynamo, and the energy transfer reach a mean equilibrium. The limits of the equilibrium are probably well controlled by second order processes. This arguably results in the observed smooth curve of the I Sequence. A high magnetic field destroys convection in the dynamo and a low magnetic field is not enough for dynamo behaviour. This contrasts with the C Sequence where the fully convective stars have different convective dynamo processes and presumably unsaturated dynamos.

The rotation periods versus mass curves of Pleiades from the Gaia data [25] are highly similar similar to Blanco 1 and are also well described by the theoretical power law models. More stars exist in the Pleiades data set and arguably the determination of the periodicity is experimentally less precise than the Blanco 1 data from NGTS. The variance of the stars in the C Sequence is broadly similar. The I Sequence follows well the $\Omega \propto M^4$ power law between $0.8 - 1.2 M_\odot$, however unlike Blanco 1 several stars lie below the I Sequence at these masses. These fast rotators may have initially possessed high angular momentum during their formation and have not had enough time to converge towards the I Sequence as witnessed in older clusters such as Praesepe described below.

3 Cluster Evolution

By comparing the young cluster data of Blanco 1 to older clusters such as Praesepe (700Myr), the evolution of clusters can be investigated. With Blanco 1 the initial angular momentum distribution dominates whereas in the longer term magnetic braking effects are expected to have a greater influence. Here we develop a theoretical model for the spin-down of the clusters and discuss the observed rotation evolution for ages ∼35 Myr to ∼950 Myr and star masses in the range $0.2 < M_\odot < 1.4$.

3.1 Model for Cluster Spin Down

A model to describe the mass dependence of the spun down state for older clusters can be developed which is not reliant on the rate of spin down and the precise magnetic field configuration. For differential rotation the amount of angular momentum ΔJ_c that must be transferred between the CZ and RZ to recouple and

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achieve solid body rotation must be

$$\Delta J_c = \frac{I_{CZ}J_{RZ} - I_{RZ}J_{CZ}}{I_{TOT}}. \tag{33}$$

Following MacGregor [17] the rate of change of angular momentum of the CZ and RZ can be expressed in terms of both a stellar spin down time τ_J and a CZ/RZ recoupling time τ_c

$$\frac{dJ_{RZ}}{dt} = -\frac{\Delta J_c}{\tau_c}, \tag{34}$$

and

$$\frac{dJ_{CZ}}{dt} = \frac{\Delta J_c}{\tau_c} - \frac{J_{CZ}}{\tau_J}. \tag{35}$$

In the wind theory of Weber[20] the spin down time is related to the mass loss rate and moment of inertia of the convective zone

$$\tau_J = \frac{3}{2}\frac{I_{CZ}}{\dot{M}r_A^2}. \tag{36}$$

The rate of change of total moment of inertia can then be written in a form similar to Eq. (1) but instead using the angular velocity of the outer convection zone which interacts with the wind.

$$\frac{dJ_{TOT}}{dt} = -\frac{2}{3}\Omega_{CZ}\dot{M}{r_A}^2 = -\frac{J_{CZ}}{\tau_J}. \tag{37}$$

First consider the case of a star rotating as a solid body on the C Sequence. Following Eq. (1) the change in angular momentum can be written discretely ΔJ as a function of the initial angular velocity $\Omega_{TOT,0}$ and total mass loss ΔM

$$\Delta J \propto -\Omega_{TOT,0}\Delta M{r_A}^2. \tag{38}$$

If the characteristic instantaneous spin-down time τ_J is approximately equal to the age of the cluster t and the stars have spun down then it can be assumed that

$$\frac{\Delta M}{M} \propto \frac{R^2}{r_A^2}, \tag{39}$$

which provides a change in momentum proportional to the angular momentum in an early cluster at $t = 0$ as

$$\Delta J \propto -J_{TOT,0}, \tag{40}$$

and thus the angular velocity of the aged cluster

$$\Omega_{TOT,t} \propto \frac{\Omega_{TOT,0}}{I_{TOT,t}} \propto \frac{J_{TOT,0}}{I_{TOT,t}I_{TOT,0}}. \tag{41}$$

Approximating $I_{TOT,t} \approx I_{TOT,0}$ then

$$\Omega_{TOT,t} \propto M^{-4.8}. \tag{42}$$

Although we have estimated $I_{TOT,t} \approx I_{TOT,0}$ it should be expected that $I_{TOT,t} < I_{TOT,0}$ due to mass loss and this power law with $x = -4.8$ probably represents a theoretical overestimate.

The above simple model can be tentatively adapted for the I Sequence with differential rotation. From Eq. (36) and Eq. (37) the change in total angular momentum can be written as

$$\Delta J_{TOT,t} \propto -\Omega_{CZ,0}\Delta M{r_A}^2. \tag{43}$$

Setting the overall momentum loss at the surface of the CZ as proportional to the total angular momentum then

$$\Omega_{CZ,0}\Delta M{r_A}^2 \propto I_{TOT,0}\Omega_{TOT,0} = J_{TOT,0}, \tag{44}$$

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which is similar to Eq. (38). If the total change in angular momentum is the sum of the change of the CZ and the RZ so

$$\Delta J_{TOT,t} = \Delta J_{CZ,t} + \Delta J_{RZ,t}, \tag{45}$$

then we can identify

$$\Delta J_{CZ,t} \propto -J_{CZ,0}. \tag{46}$$

In analogy with Eq. (41)

$$\Omega_{CZ,t} \propto \frac{\Omega_{CZ,0} I_{CZ,0}}{I_{CZ,t} I_{TOT,0}}. \tag{47}$$

Approximating $I_{CZ,t} \approx I_{CZ,0}$ and assuming as Eq. (28) $\Omega_{CZ,0} \propto M^4$ then

$$\Omega_{CZ,t} \propto M^{1.6}, \tag{48}$$

and for the RZ as before

$$\Omega_{RZ,t} \propto M^{-4.8}. \tag{49}$$

In Eq. (26) if stellar luminosity versus mass relation was assumed to be in the range $M^{3.5} - M^4$ [21] then $\Omega_{CZ,t}$ would lie in the range $M^{1.1} - M^{1.6}$ and encompass the Kawaler power law of 1.2 in Eq. (7). This model retains differential rotation on the I Sequence for older clusters rather than assuming solid body rotation as the Kawaler model.

3.2 Discussion of Cluster Evolution

In Figure 6 are shown Blanco 1 (115Myr) and 6 clusters from Gaia (NGC2547 35Myr, Pleiades 125Myr, M50 150Myr, NGC2516 150Myr, M37 500Myr, Praesepe 700Myr, NGC6811 950Myr) [25]. Although some clusters do not display stars at some mass ranges, the evolution of the C and I Sequences with time are clearly visible. To clarify, the clusters are divided into early clusters (NGC2547 35Myr, Pleiades 125Myr, M50 150Myr, NGC2516 150Myr) and late clusters (M37 500Myr, Praesepe 700Myr, NGC6811 950Myr).

The coloured lines show as before power laws of the form

$$\Omega \propto M^x, \tag{50}$$

For the early C Sequence the purple line with $x = -2.4$ as Eq. (21) provides a approximate fit to the data. For the early I Sequence are shown orange lines $x = 4$ as Eq. (28), and red lines $x = 8$ as Eq. (29) as outlined previously.

In the older clusters, such as Praesepe, the rotation periods are higher relative to the younger clusters due to spin down over time. The observed time evolution of the clusters can be arguably produced by a process of stages. For young clusters the rotation behaviour is determined by the initial angular momentum $J_{TOT,0}$ and the total moment of inertia as

$$\Omega_{TOT,0} = \frac{J_{TOT,0}}{I_{TOT}}. \tag{51}$$

If no significant braking has occurred and the average angular momentum is approximately constant across different masses for each cluster, then an average of $J_{TOT,0}$ can be used as Eq. (14). Overall $\Omega_{TOT,0}$ follows Eq. (21) but on the I Sequence differential rotation occurs so the CZ rotates at a higher rate and $\Omega_{CZ} > \Omega_{RZ}$. For the C Sequence solid body rotation occurs and $\Omega_{CZ} = \Omega_{RZ}$.

For the late clusters the I Sequence is shallower in gradient than for the younger clusters and broader, ranging from approximately $0.5 - 1.1 M_\odot$. The green line in Figure 6 shows $x = 1.2$ consistent with the Kawaler equation Eq. (7). That this provides a good fit for the older clusters suggests that significant recoupling between the CZ/RZ has occurred and differential rotation is reduced from the younger clusters.

The dynamo approach developed in Eq. (27) is proposed to model the saturated dynamo and coronal regime. However, this result is also in broad agreement for the coronal non-saturated case when γ is non zero. In the non-saturated coronal region Pizzolato [39] finds approximately $L_X/L \propto \Omega^{-2}$ and thus $\gamma = -2$. Applying $\alpha = 1/2$, nano flares $\beta = 2$ and fast rotation $\gamma = -2$ the relation is $\Omega \propto M^{1.3}$ which has an exponent close to the value of 1.2 for the magnetically braked late I Sequence and a rotational solid body.

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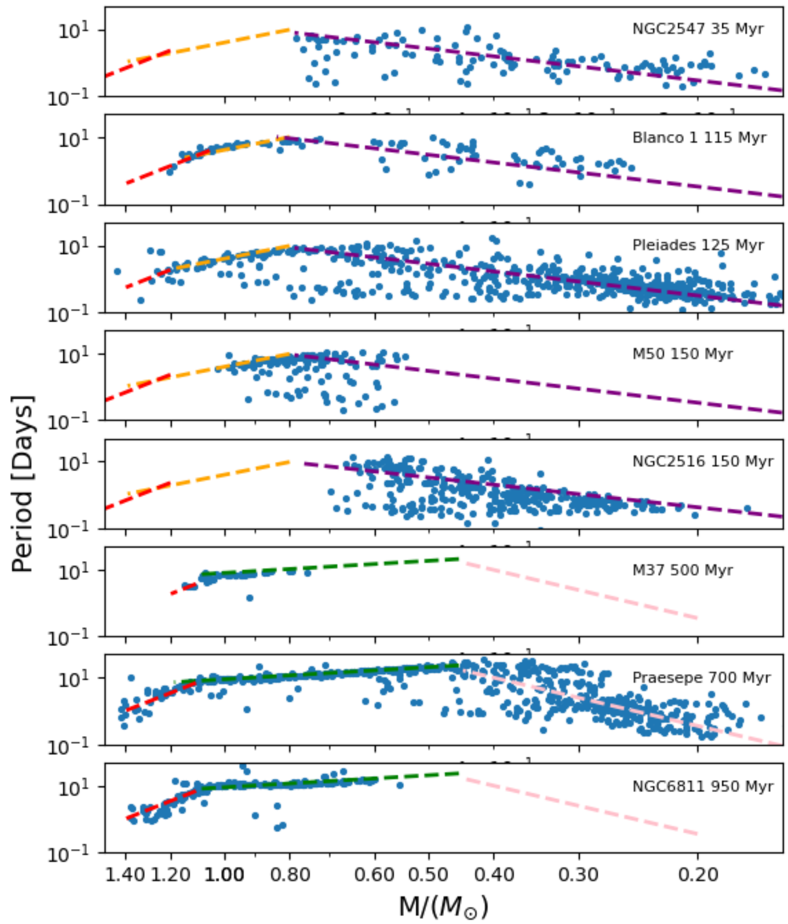

Figure 6: Rotational Period vs. Mass for clusters of different ages. The dotted lines refer to power laws of the form $\Omega \propto M^x$ where red $x = 8$, yellow $x = 4$, green $x = 1.2$, purple $x = -2.4$ and pink $x = -4.8$

This suggests that interface dynamos might still exist in the late I Sequence but with weaker differential rotation than in the early clusters. Wright [29] for the unsaturated regime takes $\gamma = -2.0$ as his base case but estimates $\gamma = -2.7$ across a wide range of stars. To modify the model to fit the late cluster data this lower level of γ would require a reduction in the effective β in the model. Wright, across all stars of different ages observed coronal saturation for rotation periods of less than 10 days whereas Pizzolato finds the saturation period decreasing with increasing mass, dropping to possibly $2 - 3$ days for higher masses. This mass dependence is consistent with the higher rotational velocities exhibiting saturated behaviour for higher masses on the early I Sequence and non saturated behaviour as they age and slow.

The C Sequence for lighter stars is however much steeper for the older clusters as a function of mass in Figure 6. This is inadequately explained by the Kawaler equation Eq. (6) for which the rate of change of angular momentum $\propto -\Omega^3$. Instead, a our spin down model developed in Eq. (42) assumes stars have spun down further from the young clusters with an additional angular momentum loss proportional to the angular momentum of the young cluster stars by the age of Praesepe. This is analogous to the model of Weber [20] but the angular velocity used is the initial angular velocity and provides the approximate relation

$$\Omega_{TOT,t} \propto M^{-4.8}. \tag{52}$$

This is plotted in Figure 6 as the pink line with $x = -4.8$. The spin down in angular velocity is effectively proportional to an additional divisive factor of moment of inertia applied to Eq. (21). This leads to a steep increase in angular velocity as a function of decreasing mass in the older clusters relative to the younger clusters.

An assumption of our model is that approximately the spin down time $t \approx \tau_J$ and is equivalent for all stars on the C Sequence. Probably the fact that stars have aged sufficiently to exceed the spin down time is sufficient for the approximation to hold.

The simple model for the C Sequence can be tentatively modified for the I Sequence as in Eq. (47) if the stellar momentum loss has the same angular velocity as the CZ and is proportional to the total angular momentum. If the initial CZ angular velocity of the early clusters is assumed to be $\Omega_{CZ,0} \propto M^4$ as above, then in the later spun down state, from Eq. (48) $\Omega_{CZ,t} \propto M^{1.6}$. This is arguably close to the Kawaler power law [18] but considering the effective spun down state and the initial rotation state provides a simpler mathematical approach than the Kawaler model.

In Figure 7 (upper) is plotted the period for various binned masses as a function of age for stars on the I Sequence for the different clusters. Lines for Skumanich relation $P \propto t^{1/2}$ are shown and appear to fit the data well, particularly over longer time periods. For the early clusters < 200Myr the data is scattered due to the complex evolution of the early rotation curves and the higher number of fast rotators lying below the I Sequence in the Gaia data. The light mass stars decelerate more rapidly than the heavier stars, which experience little deceleration since from Eq. (6) $\Omega_{TOT,t} \propto M^{1.2}$ and the rotation rate increases with mass.

Figure 7 (lower) shows, that for the lighter stars $M_\odot < 0.5$ on the C Sequence, Skumanich's law does not hold, at least in the form of Eq. (6) since the stars rotate faster with decreasing mass. The lightest stars for $M_\odot < 0.3$ there is little observable magnetic braking and they remain fast rotators even in Praesepe.

The majority of fast rotators below the I Sequence eventually migrate to the I Sequence in the late clusters due to the equilibrium energy considerations favoured by interface dynamos. In the C Sequence where possibly weaker convective dynamos exist the dispersion of periodicity remains large due to the dependence on the very initial angular momentum conditions when the stars were formed. There exists no strong dynamo process to align the stars on the C Sequence.

For higher masses $M > 1.1M_\odot$ in the the older clusters rapid rotators exist similar to the early clusters. Red lines of $x = 8$ are shown in Figure 6 consistent with Eq. (29). The continued high levels of differential rotation could be arguably due to either long CZ/RZ coupling times due to higher moments of inertia or lying above any possible saturation threshold Ω_{thres} due to thinner convective zones and thus experiencing weaker braking.

4 Conclusion

The rotational behaviour of stars in early clusters, such as Blanco 1, is determined predominately by their initial angular momentum acquired during formation and not by the longer timescale magnetic braking effects due to stellar winds.

I have outlined a simple model to describe the rotation of the Blanco 1 stars as function of mass. There exists two regimes: the C Sequence of lighter convective stars rotating as solid bodies and for heavier stars the I Sequence dominated by differential rotation and interface dynamos. On the I Sequence the low relative moment of inertia of the thinner convective zones results in observed rotation rates that can be over $10\times$ faster that of the internal radiative zone.

The mean early C Sequence angular velocity varies as the inverse of the total moment of inertia such that $\Omega \propto M^{-2.4}$. Employing energy arguments, the I Sequence differential rotation of the convective zone and saturated dynamos result in stronger mass dependence $\Omega \propto M^x$ where approximately $x = 4 - 8$ depending on the coronal heating processes leading to coronal saturation. The lighter stars on the early I Sequence can be modelled with a nanoflare coronal heating process and the heavier stars by Alfvén wave heating.

The observation of older clusters indicates that the evolution of younger clusters follows several stages. For early clusters the angular momentum and the moment of inertia determines the initial periodicity,

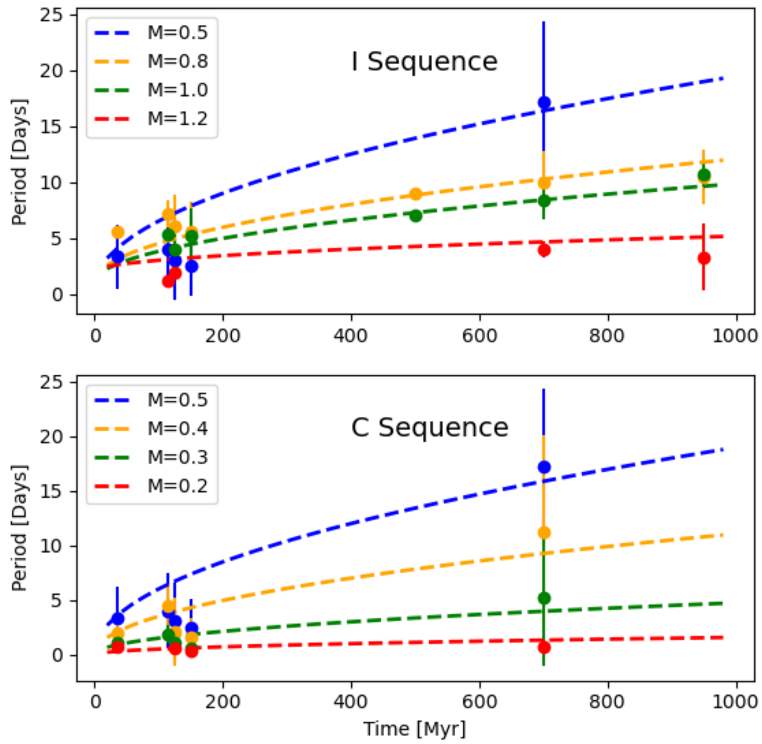

Figure 7: Rotational Period vs. Age of Cluster for different Masses. The upper Figure is for the I Sequence and lower Figure the C Sequence. The dashed lines follow Skumanich's law $P \propto t^{1/2}$.

particularly for C Sequence stars. For the early I Sequence stars the observed rotation rate is initially determined by the saturated dynamo equilibrium conditions. The ratio of magnetic to kinetic energy under magnetostrophy forces a well defined I Sequence.

Beyond this early stage, slower magnetic braking becomes the dominant factor. There is a clear flattening of the I Sequence for Praesepe and NGC6811. These stars apparently follow the modified Kawaler equation leading to a rate of change of angular momentum $\propto -\Omega^3$ and the stars slow according to Skumanich's law. The heaviest I Sequence stars in the older clusters, with very thin convective zones, slow little and observationally display fast convective zone rotation periods. These stars are rotationally similar to the earliest clusters and indicative that they retain high levels of differential rotation.

In the older clusters there is a steepening of the C Sequence for the lighter stars. The rotational deceleration is most pronounced for the lighter, convective stars. This is consistent with spin down where the change in momentum is proportional to the initial angular momentum and is well described by a simple model without specification of the form of the magnetic field or dynamo process required to achieve the spin down. This spin down process is analogous to the model of Weber [20] but largely dependent on the initial conditions of the early cluster. The model can be extended to the I Sequence and presents a similar power law to the Kawaler equation and the Skumanich relation [18] [26].

An understanding of stellar rotation and the associated magnetic fields is important for the discovery of habitable exoplanets and possible life in the universe. Even from a young age, stars in the approximate mass range $0.8 < M_{\odot} < 1.0$ possess lower rotation rates, lower magnetic fields and have balanced radiative and convection zones leading to higher stability. Planetary magnetic fields such as the Earth's, reduce the potentially harmful radiation [13] [14] caused by stellar magnetic fields and associated flare activity which are present in young stars. In an analogy, an exoplanet views a sustained major flare on the stellar surface as

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a ship views a rotating beam from a lighthouse with mean exposure proportional to the angular velocity Ω multiplied by the flare energy. If the flare energy is roughly proportional to the energy of the magnetic field B^2 then for a fast rotator the energy incident on the planet might be $\propto \Omega^2$. Thus for a planet with $M_\odot$ the radiation might be $100\times$ lower than for a slightly heavier star of $1.2M_\odot$ in a young cluster. A knowledge of the sweet spot for long stellar rotation periods and lower magnetic fields may confine the range of potentially habitable systems.

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6 Data Availability Statement

The datasets were derived from sources in the public domain from (Gillen et al. 2020) and (Godoy-Rivera et al. 2021).

7 Copyright Notice

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